CHAPTER REVIEW

Chapter 14: Matrices

QUICK CHECK

Chapter 14

Complete these exercises before trying the Practice Test for Chapter 14. If you have difficulty with a particular problem, review the indicated section.

Let $A = \begin{bmatrix} 1 & 0 & 5 \\ -3 & -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & -2 \\ 0 & 3 \\ -4 & -1 \end{bmatrix}$.

1. Explain why the sum $A + B$ is not defined. Then find $A^t - 2B$. *(Section 14-1)*
2. Find AB and BA. Are the products equal? *(Section 14-2)*
3. Show that $-A + A = O_{2\times3}$. *(Section 14-3)*
4. Matrix M at the right describes a communication network.
   - **a.** Draw a diagram to represent the network.
   - **b.** Explain how you could determine the number of ways a message can be sent using one relay.
   - **c.** Find the total number of ways a message can be sent from X using one relay. *(Section 14-4)*

   $$\text{From} \begin{matrix} X \\ Y \\ Z \end{matrix} \overset{\text{To}}{\overset{\begin{matrix} X & Y & Z \end{matrix}}{\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}}}$$

5. Suppose the matrix at the right is a transition matrix that gives the percents of customers who will switch from a national brand of applesauce to a store brand in consecutive purchases, and from a store brand to a national brand.

   $$T = \begin{matrix} \text{national} \\ \text{store} \end{matrix} \overset{\begin{matrix} \text{national} & \text{store} \end{matrix}}{\begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}}$$

   - **a.** If 70% of current customers bought a national brand of applesauce last time, what percent will buy a national brand next time?
   - **b.** Show that $S = [\,0.6 \quad 0.4\,]$ is the steady-state matrix. Explain what this means. *(Section 14-5)*
6. Let $S = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$ and $T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
   - **a.** *Complete:* $S(x, y): \to (\underline{\ ?\ }, \underline{\ ?\ })$ and $T(x, y): \to (\underline{\ ?\ }, \underline{\ ?\ })$
   - **b.** If S maps $\triangle ABC$ to $\triangle A'B'C'$, compare their areas. *(Section 14-6)*

PRACTICE TEST

Chapter 14

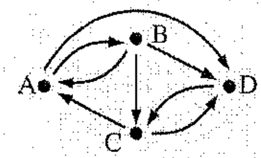
For Exercises 1–6, use matrices $A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -6 \\ 2 & -3 \end{bmatrix}$, **and** $C = \begin{bmatrix} 1 & 7 \\ 2 & -4 \\ 3 & 5 \end{bmatrix}$. **Find each matrix, if possible.**

1. $A - 2B$
2. AC
3. CB
4. C^t
5. A^{-1}
6. B^{-1}

Given the matrix equation $\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} X = \begin{bmatrix} -6 \\ 16 \end{bmatrix}$.

7. What linear system does the equation represent?
8. Solve the matrix equation for X.

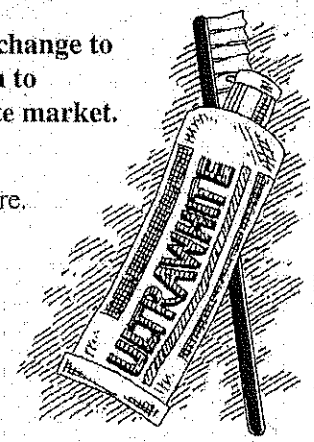
The diagram at the right shows the paths of communication between four computers in an office.



9. Write the matrix T that models this network. Label the rows and columns in alphabetical order. Then find T^2.

10. Which computer has the greatest number of two-step paths to it?

Each year 5% of the people that use UltraWhite toothpaste change to another brand and 10% of those using another brand switch to UltraWhite. Currently UltraWhite has 30% of the toothpaste market.



11. Write a transition matrix T.

12. Write a 1×2 matrix M_0 describing the current market share.

13. What will UltraWhite's market share be in 2 years?

Consider the transformation $T: (x, y) \rightarrow (2x + y, x)$.

14. Write the transformation matrix.

15. Find the images of $A(0, 0)$, $B(0, 6)$, and $C(8, 0)$.

MIXED REVIEW

Chapters 1–14

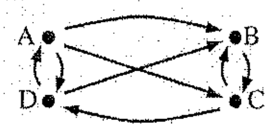
1. Solve the system $\begin{matrix} 8x - 7y = 56 \\ 5x - 4y = 20 \end{matrix}$ by using a matrix equation.

2. Consider the series $5 + 10 + \ldots$. Find the sum of the first 20 terms if the series is **(a)** arithmetic and **(b)** geometric.

3. Find the measure of $\angle A$ and the area of $\triangle ABC$, given $A(0, 0, 0)$, $B(1, 1, 1)$, and $C(1, -5, 1)$.

4. If $z = 2\sqrt{3} - 2i$, find z^8 and the two square roots of z.

5. The diagram at the right illustrates a communication network. Find the matrix that represents the number of ways messages can be sent using *at most* one relay.

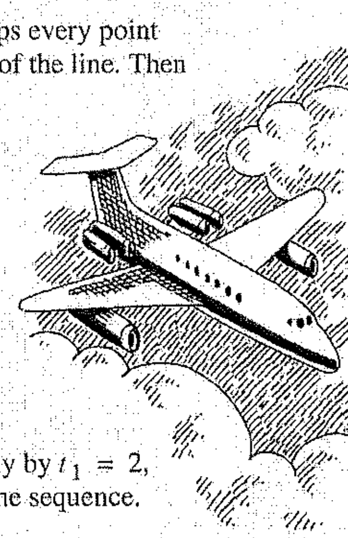


Evaluate.

6. $\sum_{n=1}^{\infty} 4 \cdot \left(-\frac{2}{3}\right)^n$ 7. $\tan(\text{Cos}^{-1} 0.8)$ 8. $\ln \sqrt[4]{e^5} + \log_4 32$

9. The transformation $T: (x, y) \rightarrow (6x - 2y, -3x + y)$ maps every point of the plane onto a line. Find the slope and an equation of the line. Then find the transformation matrix.

10. An airplane heading southwest at 500 knots encounters a wind of 50 knots blowing toward the east. Find the resultant speed and direction of the plane.



11. Let α and β be acute angles with $\sin \alpha = \frac{3}{5}$ and $\cos \beta = \frac{15}{17}$. Find $\sin(\alpha - \beta)$, $\cos 2\alpha$, and $\sin \frac{1}{2}\beta$.

12. Find $\lim_{n \to \infty} \frac{\sin n\pi}{n}$ and $\lim_{n \to \infty} \frac{n}{\sin n\pi}$.

13. Find the first 6 terms of the sequence defined recursively by $t_1 = 2$, $t_n = t_{n-1} + (2n - 1)$. Find an explicit definition for the sequence.

Name: ______________________________ **Matrices Review**

1. Which of the following is **true** for matrix addition?

a) The columns of the left matrix must equal the rows of the right matrix.

b) The matrices must have the same dimensions.

c) None of the above statements is true.

2. Given the linear equations 6x + 10y = 14 and 8x + 18y = 22,

1).Solve for x&y using Cramer's rule (where $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$)

2.) Solve it using the matrix form.

3. Evaluate the following determinant:

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$$

4. Complete: (find the determinant using minors)

$$\begin{vmatrix} 3 & 2 & 4 \\ 5 & 7 & 1 \\ 8 & 6 & 9 \end{vmatrix} = \_\_\_ \begin{vmatrix} 7 & 1 \\ 6 & 9 \end{vmatrix} + \_\_\_ \begin{vmatrix} 5 & 1 \\ 8 & 9 \end{vmatrix} + \_\_\_ \begin{vmatrix} 5 & 7 \\ 8 & 6 \end{vmatrix}$$

5. A square matrix whose main diagonal elements are 1's and whose other elements are 0's is called a(n):

____________________ matrix

6. Given matrix $A = \begin{bmatrix} 0 & -2 \\ 0 & 7 \\ -8 & 3 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 7 & 1 \\ 6 & 9 \end{bmatrix}$ Find AB.

7. If $C = \begin{bmatrix} 9 & 3 \\ 12 & 4 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 0 & -8 & 1 \\ 5 & 7 & 3 & 9 \end{bmatrix}$ what is -2C + D?

8. Given Matrix $N = \begin{bmatrix} 5 & 11 \\ -3 & -7 \end{bmatrix}$ Find N^{-1}.

9. What are the dimensions of the matrix $A = \begin{matrix} 8 & 5 & 6 & 4 \\ 0 & 4 & 13 & 1 \\ 3 & 12 & 1 & 0 \end{matrix}$?

10. Given matrix A = $\begin{bmatrix} 0 & -2 \\ 0 & 7 \\ -8 & 3 \end{bmatrix}$ and matrix B = $\begin{bmatrix} 1 & -5 \\ -3 & 1 \\ 4 & 2 \end{bmatrix}$ Find B – A.

11. If A = $\begin{bmatrix} 2 & -1 \\ 4 & 0 \\ 0 & -8 \end{bmatrix}$ what are the dimensions of the <u>transpose</u> of A:

CHAPTER REVIEW

Chapter 15: Combinatorics

QUICK CHECK

Chapter 15

Complete these exercises before trying the Practice Test for Chapter 15. If you have difficulty with a particular problem, review the indicated section.

Let P = set of voters who voted for the Republican presidential candidate. Let C = set of voters who voted for the Republican congressional candidate.

1. Describe each of these sets in words. *(Section 15-1)*

a. $P \cap C$ **b.** $\overline{P} \cup \overline{C}$ **c.** $\overline{P \cup C}$ **d.** $C \cap \overline{P}$

2. Of 2000 voters who cast ballots, 1200 voted for the Republican presidential candidate, 800 voted for the Republican congressional candidate, and 700 did not cast a vote for either Republican candidate. How many voters cast ballots for both Republican candidates? *(Section 15-1)*

3. David has twelve shows recorded on videotape, as well as shows on four different channels on TV. If David plans to watch a show on TV and then one on videotape, in how many different ways can he do this? *(Section 15-2)*

4. Find ${}_{10}P_6$ and ${}_{10}C_6$. In what ways are they used? *(Section 15-3)*

5. How many different 5-card hands can be dealt from a standard deck of cards if all the cards are to be of the same suit? *(Section 15-3)*

6. How many different ways can the letters of the word REVIVE be arranged? *(Section 15-4)*

7. In the expansion of $(a + 3)^{15}$, what is the coefficient of the term a^9? *(Section 15-5)*

PRACTICE TEST

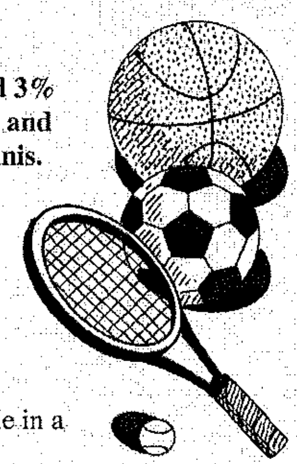
Chapter 15

Draw a Venn diagram and shade the region representing the given set of students. Let F = the set of French minors, S = the set of seniors, and B = the set of Biology majors.

1. $\overline{F} \cap S$ **2.** $F \cap S \cap \overline{B}$

3. Describe the sets given in Exercises 1 and 2.

Last year, 50% of the senior class played soccer, 52% played basketball, 26% played tennis, 4% played all three sports, and 3% did not play any of these sports. Also, 15% played both soccer and basketball, 11% basketball and tennis, and 9% soccer and tennis.



4. Draw a Venn Diagram to represent the data.

5. What percent played only one sport?

Evaluate each expression.

6. $6!$ **7.** $\dfrac{10!}{5!}$ **8.** $\dfrac{8!}{5! \cdot 3!}$

9. In how many ways can 5 different jobs be assigned to 5 people in a group of 12?

Find the number of possible arrangements of each set of items.

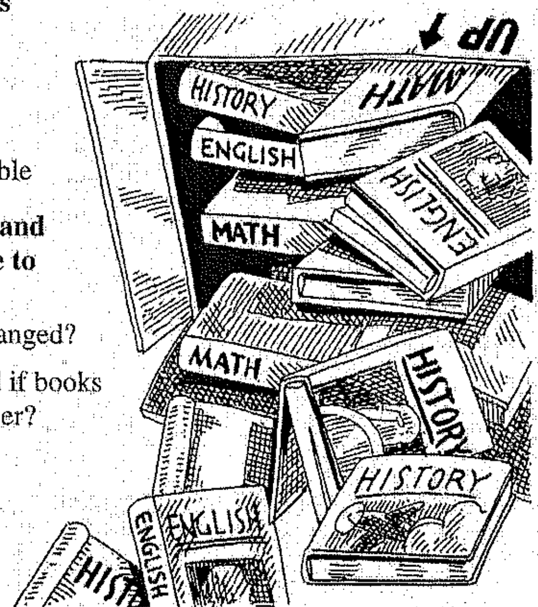
10. the letters in the word SEARCH
11. the letters in the word LEVELED
12. seven people sitting around a circular table

A box contains 5 English, 4 mathematics, and 6 history textbooks, all different. They are to be put on a shelf.

13. In how many ways can the books be arranged?
14. In how many ways can they be arranged if books of the same subject are to be kept together?

Consider the expansion of $(a - 3)^{10}$.

15. Find the 4th term of the expansion.
16. Find the term containing a^8.



MIXED REVIEW

Chapters 1–15

1. How many communication matrices are possible which describe a network of three stations (a station does not communicate with itself)?

Consider the transformation $T: (x, y) \to (3x + y, -2x)$.

2. Find the image under T of $\triangle PQR$ with vertices $P(0, 0)$, $Q(3, 4)$, and $R(0, 3)$.
3. Find the matrix T and the value of its determinant $|T|$. What information does $|T|$ provide?
4. Find the matrix T^{-1} and the transformation $T^{-1}: (x, y) \to (?, ?)$.

Find the sums. If the sum does not exist, so state.

5. $\sum_{n=2}^{20} 3n$
6. $\sum_{n=1}^{\infty} 3\left(-\frac{1}{5}\right)^n$
7. $\sum_{n=1}^{\infty} (10 + n)$

In Exercises 8–16, sketch the graph of the relation.

8. $r = 1 + \sin 2\theta$
9. $xy = -2$
10. $4y^2 + 16x^2 = 64$
11. $y = \log_2 x$
12. $\frac{x}{10} - \frac{y}{6} = 1$
13. $y = |x - 5|$
14. $y = 3 \sin 2\left(x - \frac{\pi}{4}\right)$, where $-\pi \le x \le \pi$
15. $f(n) = \left(-\frac{2}{3}\right)^n$, where $n = 0, 1, 2, \ldots$
16. $(x, y) = (2t, 1 + t^2), t \ge 0$
17. Use the binomial theorem to expand $\left(\frac{1}{4} + \frac{3}{4}\right)^6$. What is the sum of the seven resulting terms? Why?

Name: ______________________________ **MA2** **(Counting Review 1)**

1. In how many ways can a club with 20 members choose a 2-person governing council?

2. How many 3-digit numbers contain at least one digit 7?

3. A club has 12 members, 4 seniors and 8 underclassmen. How many ways can the offices of president, vice-president, secretary, and treasurer be filled if the president and vice-president must both be a senior?

4. In how many ways can you answer 25 multiple-choice questions if each question has 4 choices?

5. Of the 52 teachers at Roosevelt High School, 27 said they liked to go sailing, 25 said they like to go fishing, and 12 said they don't enjoy either recreation. How many enjoy fishing but not sailing?

6. How many 5 letter "words" begin with a vowel (a, e, i, o, or u) and end with a consonant?
(Reminder: There are a total of 26 letters.)

7. Ten people apply for 2 job positions. In how many different ways can the 2 positions be filled if the positions are all the same?

8. In how many different orders can you arrange 6 different books on a shelf?

9. In how many ways can a 6 hockey players be chosen from a group of 12 if the playing positions are unimportant?

10. In how many ways can 8 people
    a. line up in a cafeteria?

    a. sit at a round table?

CHAPTER REVIEW

Chapter 16: Probability

QUICK CHECK

Chapter 16

Complete these exercises before trying the Practice Test for Chapter 16. If you have difficulty with a particular problem, review the indicated section.

In Exercises 1–2, two letters are selected randomly, without replacement, from the letters of the word "SPAIN."

1. **a.** Write the sample space for this experiment.
   b. Find the probability that one of the letters drawn is A. *(Section 16-1)*
2. If one of the letters drawn is I, what is the probability that the other is A? *(Section 16-2)*
3. Find the probability of getting exactly 7 heads and 7 tails in 14 flips of a fair coin. *(Section 16-3)*
4. Five cards are dealt from a well-shuffled standard deck. Find the probability that the cards dealt are five consecutive cards of the same suit. *(Section 16-4)*
5. Jar A has 2 black marbles and 1 white marble. Jar B has 2 black and 3 white marbles. Without looking, a marble is drawn at random from Jar A and transferred to Jar B. Then a marble is drawn at random from Jar B. What is the probability that the second marble drawn is white? *(Section 16-5)*
6. In a game show, a contestant chooses prizes hidden behind three closed doors. One door hides a valueless "prize," another door hides a prize worth \$500, and the remaining door hides a grand prize worth \$10,000. What is the expected value (payoff) of this game? *(Section 16-6)*

PRACTICE TEST

Chapter 16

Give an example for each of the following.

1. two mutually exclusive events
2. two independent events

Three cards are selected from a well-shuffled standard deck of 52 playing cards. Find the probability of drawing each of the following.

3. three black cards
4. three face cards
5. at least one ace
6. no pairs

A pair of fair dice are rolled. Find the probability of rolling each of the following.

7. a sum less than seven
8. a pair of numbers
9. a sum of seven or eleven
10. a pair of prime numbers

The probability that Jack makes a free throw is 0.6 and the probability that April makes a free throw is 0.7. Find the probability of each of the following.

11. Each of them makes their next free throw.
12. At least one of them makes their next free throw.
13. Jack makes 7 out of his next 10 free throws.
14. April makes at least 8 out of her next 10 free throws.

A committee of 6 is to be formed from 10 students and 8 faculty members. Find the probability of each of the following.

15. Exactly 2 students are on the committee.

16. At most 4 students are on the committee.

Jar A contains 4 red and 3 white marbles and Jar B contains 3 red and 2 white marbles. A die is rolled. If the number rolled is greater than 2, a marble is selected from Jar B, otherwise a marble is selected from Jar A.

17. Draw a tree diagram that displays this information.

18. Find the probability that the marble is red.

19. The marble selected is red. Find the probability that the marble came from Jar A.

For a particular game, the probability of winning \$50 is 0.2 and the probability of winning \$10 is 0.6.

20. How much would you expect to win in this game?

21. What would be a fair price to play this game?

MIXED REVIEW

Chapters 1–16

1. a. Copy and complete the table of values below for binomial probabilities with $n = 6$ and $p = 0.4$. Round your answers to the nearest hundredth.

r	0	1	2	3	4	5	6
$p(r) = {}_6C_r(0.4)^r(0.6)^{6-r}$							

b. Graph the points $(r, p(r))$ and connect them with a smooth curve.

2. Repeat Exercise 1, this time with $n = 6$ and $p = 0.5$.

3. Compare and contrast the graphs from Exercises 1 and 2. Consider their slopes, symmetry, maxima, and minima.

4. Find the sum $\sum_{i=1}^{10} \left(\frac{3i}{20}\right)^2$.

Solve these trigonometric equations.

5. $\sin x - \cos^2 x \sin x = -1$

6. $\sin 3x \cos 2x - \cos 3x \sin 2x = 0.5$

7. $2 \sin^2 \frac{1}{2}x = 2 + \cos x$

8. $\cos (45° - \theta) = 3 \cos \theta$

Find the limit of the sequence or state that the limit does not exist.

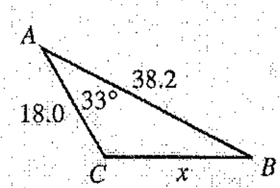
9. $t_n = {}_nC_3, n = 3, 4, 5, \ldots$

10. $t_1 = 100, t_n = \left(\frac{1}{2}t_{n-1} + 20\right)$

11. $t_n = \frac{18}{3^n}, n = 1, 2, 3, \ldots$

12. $t_n = \left(1 + \frac{0.06}{n}\right)^n, n = 1, 2, 3, \ldots$

For Exercises 13 and 14, refer to $\triangle ABC$ at the right.



13. Solve for x.

14. Find the measures of $\angle C$ and $\angle B$ to the nearest tenth of a degree.

Name: ____________________ **MA2 (Probability Review)**

1. In how many ways can 4 **different** prizes be awarded to three people picked from a group of 60?

2. How many different arrangements are possible of the letters in the word "TOMATO"? .

3. Give the seventh row of Pascal's triangle.

4. Expand $(x+y)^4$ by binomial theorem and Pascal's triangle .

5. If a card is randomly selected from a standard deck of 52, the events selecting a face card and selecting a club are independent, not independent or mutually exclusive?

6. From a box containing 5 red balls and 3 yellow balls, 2 balls are randomly picked, one after the other and without replacement. Find the probability that one is red and one is yellow.

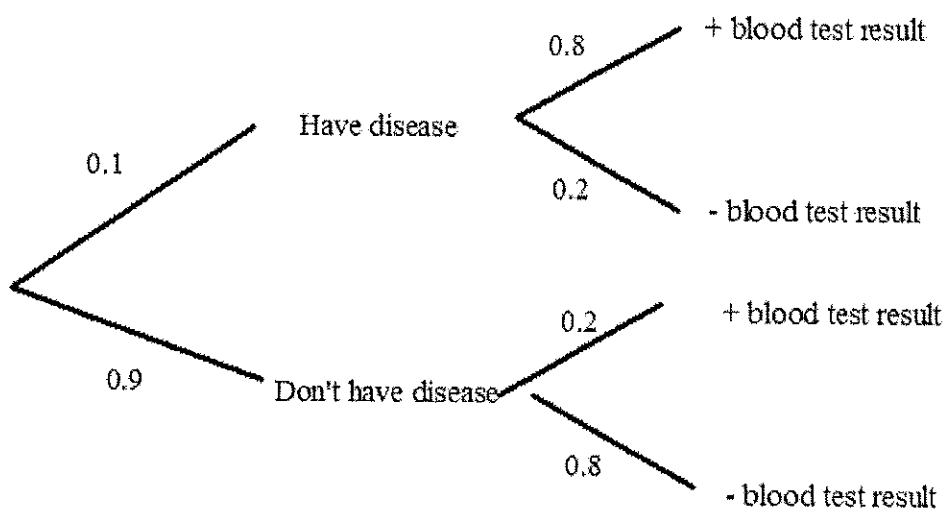
7. A card is randomly selected from a standard deck of cards. Let event A="selecting a queen" and event B= "selecting a face card". Which of the following probabilities would be equal to 1/3?

   P(A/B) OR P(B/A)

8. If 20% of the population is married, what is the probability that 2 out of 5 people selected at random, are married.

9. Four marbles are picked at random from a bag containing 4 red marbles and 5 white marbles. What is the probability of selecting exactly 3 red marble.

10.



Calculate P(have disease|negative blood test result)

11. A wheel divided into four equal areas A, B, C D is spun four times. You put your bet down on one of the areas. If the wheel lands on the same area all four times, you win $20. If the wheel lands on your area three times, you win $2. Otherwise you lose your bet.

a.) If your bet is $1 to play the game, what is your expected win or loss?

b.) Is the game fair? Why or why not? If not, how can you change the payoffs (other than making them $0) to make it fair.