NAME: ____________________ PERIOD: ___

DATE: _______________ MATH ANALYSIS 2

MR. MELLINA

CHAPTER 14: MATRIX OPERATION

Sections:

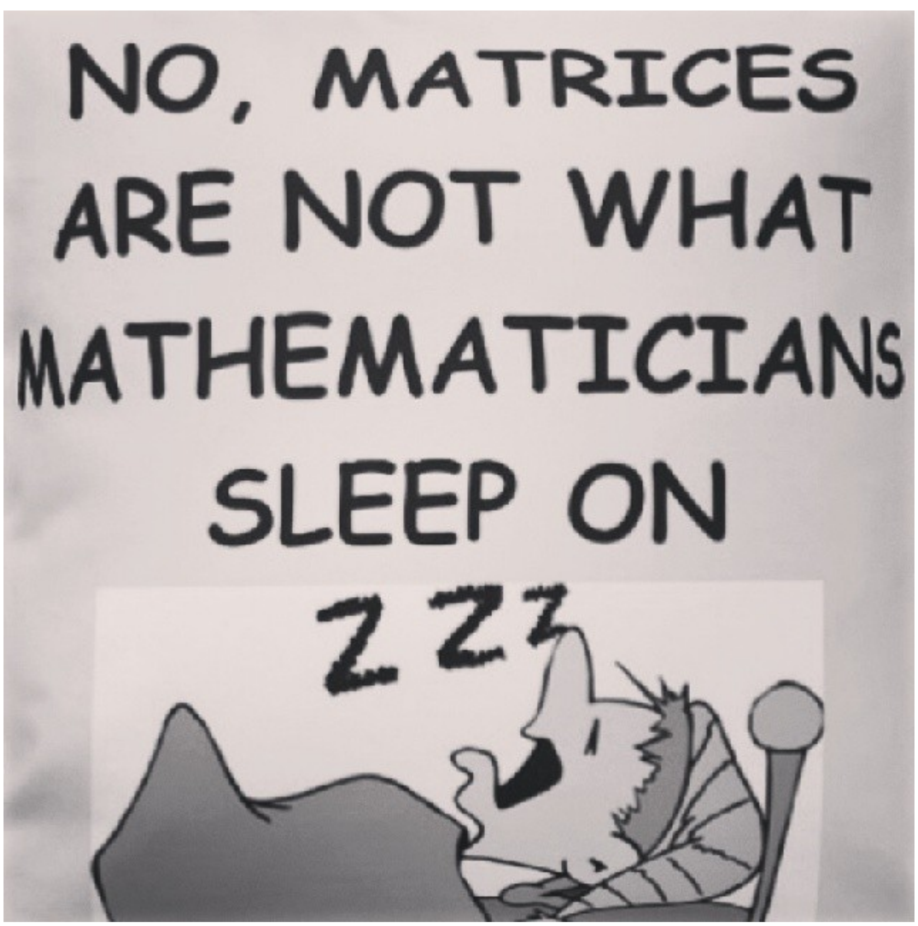
- *14.1 Matrix Addition and Scalar Multiplication*
- *14.2 Matrix Multiplication*
- *14.3 Applying Matrices to Linear Systems*

HW Sets

***Set A** (Section 14.1) Pages 521 & 522, #'s 2-16 even, 20.*

***Set B** (Section 14.2) Pages 526-529, #'s 2-20 even.*

***Set C** (Section 14.3) Pages 534 & 536, #'s 2-24 even.*



14.1 MATRIX ADDITION AND SCALAR MULTIPLICATION (PAGE 517)

14.1 Warm Up!

Let **u** = (1, 5, -2) and **v** = (3, -9, 0). Find:

a. **u** + **v** b. **u** – **v** c. $\frac{1}{3}\mathbf{u}$ d. **v** – 2**u**

Example 1:

An automobile dealer sells four different models whose fuel economy is shown below. Find the following

	Sports car	Sedan	Station Wagon	Van
Miles per gallon for city driving	17	22	17	16
Miles per gallon for highway driving	23	30	24	19

a. E

b. Dimensions of E

c. Label the rows and columns of *E*

d. What does Matrix *E* represent?

e. Find E^t

f. What does E^t represent?

g. Suppose that the Environmental Protection Agency (EPA) mandates that all of these fuel performance figures must increase 10% by the year 1998. Find a new matrix that represents the totals after the 10% increase.

Matrices

A matrix is a rectangular _________ of numbers enclosed by _____________ (named by a capital letter)

Each number in a matrix is called an ______________.

The number rows and columns is called the _________________ of the matrix.

Ex: $E_{2\times4}$ is matrix E with 2 rows and 4 columns

_______________: Interchanging the rows & columns. Denoted: ____.

_________ ________: Multiplying each element by a scalar.

Matrix Addition/Subtraction: add/subtract each __________________ ____________.

Example 2

Given: $A = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, B = \begin{bmatrix} 2 & 5 \end{bmatrix}, C = \begin{bmatrix} 5 & 2 \end{bmatrix}, D = \begin{bmatrix} 2 & \frac{10}{2} \end{bmatrix}$

a. Give the dimensions of each matrix

b. Which matrices are transposes of each other?

c. Which matrices are equal to each other?

Example 3

Given: $M = \begin{bmatrix} 3 & 1 & 5 \\ 4 & 0 & -2 \end{bmatrix}$, find

a. M^t

b. 2*M*

c. -3*M*

Example 4:

Let $A = \begin{bmatrix} 3 & 8 & 1 \\ 4 & 0 & -3 \\ -2 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 9 \\ 4 & -6 & -5 \\ 0 & 7 & 2 \end{bmatrix}$, find

a. $A + B$

b. $A - 2B$

c. $A + B^t$

Example 5:
The Handy Hardware Company has two locations, one downtown and one at the mall. During April, the downtown store sold 31 of the lowest-priced lawn mowers, 42 of the medium-priced ones and 18 of the highest-priced mowers. Also during April, the mall store sold 22 of the lowest-priced mowers, 25 of the medium-priced mowers, and 11 of the highest-priced mowers.

a. Represent this information in an April sales matrix *A*.

b. Do the dimensions of your matrix *A* represent a "mower-type by location" matrix or a "location by mower-type" matrix?

c. Suppose that during May, the Handy downtown store sold 28 of the lowest-priced mowers, 29 of the medium-priced ones and 20 of the highest-priced ones, and that the mall store sold 20 lowest-priced ones, 28 medium-priced ones, and 9-highest priced ones. Represent this information in a May Matrix *M* that has the same dimensions as *A*.

d. Find *A* + *M* and describe what this matrix sum tells you.

e. If the manager of the Handy stores expects next year's lawn mower sales to rise about 8%, about how many highest-priced mowers does the manager expect to sell at the downtown store next April? If you were the manager, would you round your calculations up or down? What scalar multiple of matrix *A* would assist you in planning next April's sales?

Example 6: Independent work *

Simplify

a. $\begin{bmatrix} -3 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

b. $\begin{bmatrix} 8 & 1 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$

c. $\begin{bmatrix} 8 & 2 & -2 \\ -3 & 1 & 14 \end{bmatrix} + \begin{bmatrix} 12 & 3 & 10 \\ 0 & 0 & -6 \end{bmatrix}$

d. $8\begin{bmatrix} 5 & -2 \\ 4 & 0 \end{bmatrix}$

e. $2\begin{bmatrix} 3 & 0 \\ -4 & 1 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} -2 & -2 \\ 3 & 0 \\ 6 & 11 \end{bmatrix}$

f. $\begin{bmatrix} 1 & -4 & 2 \\ 5 & 0 & 7 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ 9 & 10 \\ -1 & -2 \end{bmatrix}$

Example 7: Independent work **

Let $A = \begin{bmatrix} 3 & 2 & 4 & 8 \\ 0 & 1 & 8 & -2 \\ 3 & 7 & 9 & 1 \end{bmatrix}, B = \begin{bmatrix} 8 & 1 & 4 \\ 3 & 0 & 7 \\ -2 & 5 & 11 \\ 6 & 4 & -3 \end{bmatrix}, C = \begin{bmatrix} 9 & 0 & 5 \\ 4 & 3 & 1 \\ 7 & 12 & 2 \\ 6 & 0 & 3 \end{bmatrix}$, find

a. A^t

b. $A^t + B$

c. $3C - A^t$

Example 8: Independent work ***

Find the values of the variable for which the given statement is true

a. $\begin{bmatrix} 2x-3y & 3 \\ -7 & 24-y \end{bmatrix} = \begin{bmatrix} -2 & w \\ -7 & 4x \end{bmatrix}$

b. $\begin{bmatrix} y & 5 \\ 2-c & 3x+\frac{y}{3} \end{bmatrix} = \begin{bmatrix} 36-9x & 2a+1 \\ -8 & 12 \end{bmatrix}$

Example 9: Independent work ***

a. Is it true that $(A+B)^t = A^t + B^t$ for any matrices *A* and *B* that have the same dimensions? Give an example to support your answer.

b. Give a real-world example of two matrices that have the same dimensions and whose sum makes no sense. Explain why the resulting matrix is not meaningful.

14.2 MATRIX MULTIPLICATION (PAGE 523)

14.2 Warm Up!

a. If an operation * is commutative, then a * b = ____________.

b. If $b * h = \frac{1}{2}bh$ is * a commutative operation?

c. If $r * h = \frac{1}{3}\pi r^2 h$, is * a commutative operation?

Example 1:
Suppose a teacher calculates your test average for the term by using a formula that counts, or weighs, each of your five tests a certain percentage of your grade, as shown in matrix *W* below. Notice that the weights must add up to 100%

	Test 1	*Test 2*	(*midterm*) *Test 3*	*Test 4*	(*final exam*) *Test 5*
weight	15%	15%	25%	15%	30%

$$weight\ [15\% \quad 15\% \quad 25\% \quad 15\% \quad 30\%] = W$$

How could you use these weights to find the weighted score, or *weighted average*, of students *A*, *B*, and *C*, whose test scores are given in matrix *T*.

	A	*B*	*C*
Test 1	82	92	74
Test 2	85	88	68
Test 3	78	95	73
Test 4	75	85	82
Test 5	84	94	81

$= T$

To calculate a student's weighted score (or *weighted average*), you must ___________ each test score by its weight and then __________ the products.

Multiplying Matrices

The product of two matrices A and B is defined only when the number of columns in A is _________ to the number of rows in B. If A is an $m \times n$ *matrix and matrix B is an* $n \times p$ *matrix, then the product AB is an* $m \times p$ *matrix.*

Ex: You can only multiply matrices that fit the following dimensions

Dimensions of Resulting Matrices

Example 2

You are given the dimensions of matrices *A* and *B.* Is *AB* defined? Is *BA* defined? Give the dimensions of each possible product.

a. $A_{3\times5}$ $B_{5\times2}$

b. $A_{7\times6}$ $B_{2\times7}$

c. $A_{1\times4}$ $B_{3\times1}$

Example 3

Find the product

a. $\begin{bmatrix} 1 & 2 & 0 \\ 3 & -5 & 2 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 0 & 4 \\ 1 & -1 \end{bmatrix}$

b. $\begin{bmatrix} 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 0 & -4 \\ -2 & 7 \end{bmatrix}$

Example 4:

Machine I and Machine II produce X, Y, and Z at the hourly rate given in matrix *H*. Matrix *D* gives the number of hours each machine runs during the week.

$$\begin{array}{c} \\ X \\ Y \\ Z \end{array} \begin{array}{c} \begin{array}{cc} I & II \end{array} \\ \begin{bmatrix} 3 & 2 \\ 5 & 4 \\ 1 & 2 \end{bmatrix} \end{array} = H \qquad \begin{array}{c} \\ I \\ II \end{array} \begin{array}{c} \begin{array}{ccccc} M & T & W & Th & F \end{array} \\ \begin{bmatrix} 8 & 8 & 8 & 7 & 7 \\ 6 & 10 & 12 & 11 & 9 \end{bmatrix} \end{array} = D$$

a. Give the dimensions of *H*, *D*, and *HD*.

b. Find *HD*. What information does *HD* give?

c. How many Y items are produced on Monday? How many Z items are produced on Thursday?

Example 5: Independent work *

You are given the dimensions of matrices *A* and *B.* Is *AB* defined? Is *BA* defined? Give the dimensions of each possible product.

a. $A_{5\times 7}$, $B_{7\times 3}$

b. $A_{2\times 4}$, $B_{3\times 4}$

c. $A_{3\times 4}$, $B_{4\times 3}$

Example 6: Independent work *

Find each matrix product, if it is defined.

a. $\begin{bmatrix} 4 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 5 \\ 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 4 & -4 \\ 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 8 & -10 \\ 0 & 3 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} -2 \\ -9 \\ 1 \end{bmatrix}$

d. $\begin{bmatrix} 9 & -4 & 4 \\ 2 & -1 & -6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & -3 \\ 3 & 5 & 2 \end{bmatrix}$

e. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

Example 7: Independent work **

Matrix *S* gives you the number of three types of cars sold in March by two car dealers, and matrix *P* gives the profit for each type of car sold.

$$\begin{array}{r} \\ \\ \textit{compact} \\ \textit{mid-size} \\ \textit{full-size} \end{array} \begin{array}{c} \textit{dealer} \\ 1 \quad 2 \\ \begin{bmatrix} 18 & 15 \\ 24 & 17 \\ 16 & 20 \end{bmatrix} \end{array} = S \qquad \begin{array}{c} \quad \textit{compact} \quad \textit{mid-size} \quad \textit{full-size} \\ \text{Profit}\begin{bmatrix} \$400 & \$650 & \$900 \end{bmatrix} = P \end{array}$$

a. Which matrix is defined, *SP* or *PS*?

b. Find this matrix and interpret its elements.

Example 8: Independent work **

Matrix *P* gives the monthly production schedule for three models of calculators. Matrix *M* gives the number of components needed to construct each model. Matrix *R* gives the number of relays needed for each component.

$$\begin{array}{r} \\ \text{scientific} \\ \text{business} \\ \text{graphing} \end{array} \overset{\text{Components}}{\begin{array}{ccc} A & B & C \\ \end{array}} \begin{bmatrix} 2 & 1 & 2 \\ 2 & 1 & 4 \\ 2 & 5 & 6 \end{bmatrix} = M \qquad \begin{array}{r} \\ \text{scientific} \\ \text{business} \\ \text{graphing} \end{array} \begin{array}{ccc} \text{Jan.} & \text{Feb.} & \text{Mar.} \end{array} \begin{bmatrix} 500 & 600 & 600 \\ 200 & 200 & 200 \\ 100 & 300 & 400 \end{bmatrix} = P \qquad \overset{\textit{Components}}{\begin{array}{ccc} A & B & C \end{array}} \begin{array}{r} \textit{relay } x \\ \textit{relay } y \end{array} \begin{bmatrix} 5 & 3 & 2 \\ 6 & 4 & 3 \end{bmatrix} = R$$

a. Explain why the product *PM* is defined, but not meaningful.

b. Find M^tP. Use your answer to tell how many A components will be needed in March.

c. Explain why *MR* is not defined.

d. Find RM^t. What information does this product give?

e. Find RM^tP. What information does this product give?

Example 9: Independent Work ***

(*Complete example 6e before trying this example*)

Consider the price matrix *P*, shown below, which gives the costs for flying first class, business class, and coach class on four different airlines, A, B, C, and D.

$$\begin{array}{c} \\ first \\ business \\ coach \end{array} \begin{array}{c} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} \\ \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{bmatrix} \end{array} = P$$

Suppose that the prices in matrix *P* are going to increase as follows:

Airline A: increase all prices by 5%
Airline B: increase all prices by 4%
Airline C: increase all prices by 6%
Airline D: increase all prices by 2%

Write a 4 x 4 increase matrix *N* such that the product *PN* is the new price matrix; that is, each element of *PN* gives the new price for each class on a particular airline. Show the matrix *PN*.

14.3 APPLYING MATRICES TO LINEAR SYSTEMS (PAGE 530)

14.3 Warm Up!

Let $A = \begin{bmatrix} 3 & -1 \\ -4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$, find

a. $(A)(A)$ b. $(B)(B)$ c. $(A)(A) - (B)(B)$

	Properties of Real Numbers	*Properties of Matrices*
Property	Let *a, b,* and *c* be real numbers	Let *A*, *B*, and *C* be $m \times n$ matrices
Commutative	*a* + *b* = *b* + *a* *ab* = *ba*	
Associative	(a + b) + c = a + (b + c) (ab)c = a(bc)	
Distributive	a(b + c) = ab + ac = ba + ca = (b + c)a	

Special Matrices

Zero matrix: any matrix whose elements are all ________. Denoted

Ex:

Square matrix: any matrix having the same number of rows as _____________.

Ex:

Identify matrix: any square matrix whose main diagonal (extends from the upper ______ to lower _________) elements are ____ and whose other elements are ____. Denoted

Ex:

Additive Inverse: the additive inverse of a matrix is the matrix in which each element is the ______________ of its corresponding element in the first matrix. Denoted

A matrix when added to its additive inverse matrix will sum to the ______ matrix.

Ex:

	Properties of Real Numbers	*Properties of Matrices*
Properties for Addition	Let a be any real number	Let A be any $m \times n$ matrix. Let O be the $m \times n$ zero matrix
Identity	$a + 0 = 0 + a = a$	
Inverse	a + (-a) = -a + a = 0	

Multiplicative Inverse of a 2 x 2 Matrix

A matrix when multiplied by its multiplicative inverse will result in the __________ matrix.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ *then* $A^{-1} = \frac{1}{|A|}\begin{bmatrix} \quad & \quad \end{bmatrix}$, *where* $|A| =$

If $|A| = 0$ *then* A^{-1}

Example 1

Given: $A = \begin{bmatrix} 5 & 4 \\ 2 & 3 \end{bmatrix}$, find A^{-1}. Check your answer by finding $A \cdot A^{-1}$.

Example 2

Let *A*, *B*, *C,* and *X* be $n \times n$ matrices, where A^{-1} exists. Solve the following matrix equation for X

a. $AX + B = C$

b. $\begin{bmatrix} 7 & -2 \\ -4 & 1 \end{bmatrix} X - \begin{bmatrix} 3 & -5 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 2 & -3 \end{bmatrix}$

Example 3:
Use matrices to solve the given system of equations:

a. $5x + 4y = -2$
$2x + 3y = -5$

Example 4:
Rewrite the following the following system as a single matrix equation.

a. $3x_1 - 5x_2 + 4x_3 + 7x_4 = 12$
$6x_1 + 7x_2 - 8x_4 = 29$
$5x_1 + 9x_2 - 7x_3 - 5x_4 = -52$
$x_2 - 5x_3 + 9x_4 = 41$

Example 5: Independent work *

Let $A = \begin{bmatrix} 8 & -3 \\ -5 & 2 \end{bmatrix}, B = \begin{bmatrix} 7 & -1 \\ 6 & 2 \end{bmatrix}, C = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 3 & 0 \end{bmatrix}, D = \begin{bmatrix} 3 & 6 \\ 4 & 8 \end{bmatrix}$

a. Find the additive inverse of *B* and of *C*.

b. Find the multiplicative inverse of *A* and of *B*.

c. Explain why *C* and *D* do not have multiplicative inverses.

d. Find A^2. To find A^3, do you calculate $A^2 \cdot A$ or $A \cdot A^2$? Explain

Example 6: Independent work **

Let $M = \begin{bmatrix} 3 & 4 \\ 6 & 7 \end{bmatrix}, N = \begin{bmatrix} 5 & 11 \\ -3 & -7 \end{bmatrix}, C = \begin{bmatrix} 9 & 3 \\ 12 & 4 \end{bmatrix}, D = \begin{bmatrix} 2 & 0 & -8 & 1 \\ 5 & 7 & 3 & 9 \end{bmatrix}$, Find the specified matrices. If a matrix does not exist, write *not defined*.

a. M^{-1}

b. MM^{-1}

c. $-C$

d. C^{-1}

e. $-\frac{1}{3}C$

f. $-2D + C$

Example 7: Independent work **

Solve each matrix equation for *X*. Assume any required inverse matrix exists.

a. $DX = E$

b. $XD = E$

c. $X - XA = D$

Example 8: Independent work **

Solve each matrix equation for *X*

a. $X + 3\begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 12 & 7 \\ 0 & -8 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} X = \begin{bmatrix} 9 & 3 \\ 9 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} X + \begin{bmatrix} 2 & 7 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$

Example 9: **Independent Work *****

Write each system of equations as a single matrix equation. Solve the system using the method shown in Example 3

a. $8x - 2y = -14$
$12x + 3y = 9$

b. $8x - 2y = 38$
$12x + 3y = 39$

c. (In Calculator)

$$3x + 5y - 9z = 26$$
$$4x + 7y + 2z = -7$$
$$6x - 9y - 8z = 3$$

Example 10: Independent Work ***

A dietician wants to combine four foods (I, II, III, and IV) to make a meal having 78 units of vitamin A, 67 units of vitamin B, 146 units of vitamin C, and 153 units of vitamin D. The matrix below gives the vitamin content per ounce of each food. How many ounces of each food should be included in the meal? Begin by writing the unknowns in a 4 x 1 matrix. Set up a matrix equation and then solve, giving answers to the nearest ounce.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} I & II & III & IV \end{array} \\ \begin{bmatrix} 3 & 2 & 2 & 6 \\ 2 & 3 & 5 & 0 \\ 8 & 6 & 4 & 7 \\ 5 & 5 & 8 & 6 \end{bmatrix} \end{array}$$

Example 11: Independent Work ***

Suppose the points (4, 11), (-6, -9), and (8, 61) are known to lie on the parabola $y = ax^2 + bx + c$. Write three equations in terms of *a, b,* and *c*. Express this system as a single matrix

MATH ANALYSIS II

Review – Matrix quiz 14.1 – 14.3

$$A = \begin{bmatrix} 5 & 10 & 1 \\ -3 & 2 & 0 \\ -1 & -2 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 7 \\ 3 & -2 \\ 5 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 4 \\ 2 & -3 \\ 6 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} -3 & 5 \\ 2 & -1 \end{bmatrix}$$

Find:

$AC =$

$BA =$

$C^tB =$

$B + C =$

$CD =$

$D^{-1} =$

$DD^{-1} =$

Solve for matrix X:

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} X + \begin{bmatrix} 3 & 1 & -2 \\ 4 & 7 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 3 & -5 \\ 2 & -4 & 6 \end{bmatrix}$$

Solve using matrices:

$$3x - 4y = 17$$
$$2x + 5y = 3$$

Identity matrices – know what it is, what it means, when it shows up

Inverse matrices – know the conditions necessary for it to exist. Be able to give examples of 2 x 2 and 3 x 3.

Matrix Word Problems ex. p. 529 #20

Chapter Test p. 560-561 # 1 – 5

14.1-14.3 Quiz Review Answers

$$AC = \begin{bmatrix} 26 & -9 \\ 4 & -18 \\ 20 & 6 \end{bmatrix}$$

BA is undefined

$$C^tB = \begin{bmatrix} 36 & 8 \\ 0 & 36 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 1 & 11 \\ 5 & -5 \\ 11 & 3 \end{bmatrix}$$

$$CD = \begin{bmatrix} 8 & -4 \\ -12 & 13 \\ -16 & 29 \end{bmatrix}$$

$$D^{-1} = \frac{1}{-7}\begin{bmatrix} -1 & -5 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & \frac{5}{7} \\ \frac{2}{7} & \frac{3}{7} \end{bmatrix}$$

$$DD^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$X = \frac{1}{-10}\begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} -4 & 2 & -3 \\ -2 & -11 & 3 \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} 2 & 35 & -12 \\ 12 & -30 & 18 \end{bmatrix} = \begin{bmatrix} -.2 & -3.5 & 1.2 \\ -1.2 & 3.0 & -1.8 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -4 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 17 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{23}\begin{bmatrix} 5 & 4 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 17 \\ 3 \end{bmatrix} = \frac{1}{23}\begin{bmatrix} 97 \\ -25 \end{bmatrix} \Rightarrow x = \frac{97}{23}, y = \frac{-25}{23}$$