Warm Up: Evaluate each limit. Note the strategy you are using to evaluate the limit.

1. $\lim_{x\to 2}\dfrac{x^2+x-6}{x-2}$

2. $\lim_{x\to\infty}\dfrac{3x}{x^2-3}$

3. $\lim_{x\to 0}\dfrac{\sin x}{x}$

4. $\lim_{x\to 0}\dfrac{e^{2x}-1}{x}$

In each example above, the limit takes on an _____ form (either $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$). Notice that only the first example was evaluated using algebraic techniques. Now that we know how to find a derivative, we have an alternate way to evaluate limits such as these using the following rule:

L'Hôpital's (aka L'Hospital's) Rule

Suppose that *f* and *g* are differentiable functions on the open interval *I* containing *a*, but not necessarily at $x = a$. Also suppose that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ are both 0 or are both $\pm\infty$ and $g'(x) \neq 0$ on *I* if $x \neq a$. Then,

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$ when the latter exists.

In other words, if a limit yields the indeterminate forms or , then we can find the limit of the quotient of the derivatives of the functions to evaluate that limit.

Remember to check all three parts of the hypothesis BEFORE applying this rule!

Example 1: Identify the indeterminate form for each limit. Then, use L'Hôpital's Rule to evaluate.

a) $\lim_{x\to 2}\dfrac{x^2+x-6}{x-2}$

b) $\lim_{x\to\infty}\dfrac{3x}{x^2-3}$

c) $\lim_{x\to 0}\frac{\sin x}{x}$

d) $\lim_{x\to 0}\frac{e^{2x}-1}{x}$

Example 2: Determine whether or not L'Hopsital's Rule applies. If so, use it to evaluate the limit. If not, explain why not.

a) $\lim_{x\to\infty}\frac{e^{-x}}{x}$

b) $\lim_{x\to\infty}\frac{x-\sin x}{x+\sin x}$

Calculus Practice 1.1C3: Limits Using L'Hospital's Rule 1a

Evaluate each limit using L'Hôpital's Rule.

1) $\lim_{x\to\infty} \frac{\ln x}{x}$

A) 0 B) ∞
C) −2 D) 1

2) $\lim_{x\to 0^+} \frac{4\ln\tan x}{\ln\sin x}$

A) 0 B) −2
C) ∞ D) 4

3) $\lim_{x\to\infty} \frac{2x}{e^{2x}}$

A) −1 B) ∞
C) 2 D) 0

4) $\lim_{x\to -1} \frac{2\left(x^2-1\right)}{\ln x^2}$

A) 0 B) 2
C) 1 D) ∞

5) $\lim_{x\to 0} \frac{e^x-1}{5x}$

A) $\frac{1}{5}$ B) ∞
C) 0 D) 1

6) $\lim_{x\to\infty} \frac{e^x}{2x}$

A) 1 B) 0
C) −3 D) ∞

7) $\lim_{x\to -1} \frac{4\ln x^2}{x^2-1}$

A) 4 B) ∞
C) 0 D) −1

8) $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$

A) 3 B) $\frac{1}{6}$
C) ∞ D) 0

9) $\lim_{x\to 0} \frac{4\left(e^x-e^{-x}\right)}{\sin(2x)}$

A) −2 B) 4
C) 0 D) ∞

10) $\lim_{x\to 2} \frac{\sqrt{x+14}-4}{x-2}$

A) 4 B) ∞
C) $\frac{1}{8}$ D) 0

11) $\lim_{x \to 0} \frac{e^{5x} - 1}{\sin(5x)}$

A) 1 B) ∞
C) 5 D) 0

12) $\lim_{x \to 1} \frac{2(x - 1)}{\ln x}$

A) 2 B) 0
C) ∞ D) 4

13) $\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$

A) ∞ B) 0
C) 1 D) 5

14) $\lim_{x \to 0^+} \frac{\ln \sin x}{\ln \tan x}$

A) 1 B) ∞
C) 0 D) -1

15) $\lim_{x \to 1} \frac{2 \ln x}{x - 1}$

A) 2 B) 0
C) ∞ D) -4

16) $\lim_{x \to 0} \frac{4x}{\ln(x + 1)}$

A) ∞ B) -4
C) 4 D) 0

17) $\lim_{x \to \infty} \frac{e^x}{2x^2}$

A) 0 B) ∞
C) -3 D) 1

18) $\lim_{x \to 0} \frac{2(e^x - 1 - x)}{x^2}$

A) ∞ B) 2
C) 0 D) 1

19) $\lim_{x \to 0} \frac{x^2}{e^x - 1 - x}$

A) 2 B) 0
C) 1 D) ∞

20) $\lim_{x \to \infty} \frac{2x^2}{e^{2x}}$

A) ∞ B) -3
C) 2 D) 0

Calculus Practice 1.1C3: Limits Using L'Hospital's Rule 1b

Evaluate each limit using L'Hôpital's Rule.

1) $\lim_{x \to 0} \frac{\sin(3x)}{\sin(5x)}$

2) $\lim_{x \to 0} \frac{\tan(3x)}{x}$

3) $\lim_{x \to 0^+} \frac{\ln \sin x}{\ln \tan x}$

4) $\lim_{x \to 0} \frac{e^x - e^{-x}}{x}$

5) $\lim_{x \to 1} \frac{2 \ln x}{x - 1}$

6) $\lim_{x \to -1} \frac{3 \ln x^2}{x^2 - 1}$

7) $\lim_{x \to -3} -\frac{x^2 - x - 12}{x + 3}$

8) $\lim_{x \to 0} \frac{3\left(e^x - e^{-x}\right)}{\sin x}$

9) $\lim_{x \to \infty} \frac{\ln x}{2x}$

10) $\lim_{x \to 0} \frac{e^{2x} - 1}{3x^3}$

11) $\lim_{x \to 0} \frac{e^{3x} - 1}{\sin(3x)}$

12) $\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25}$

13) $\lim_{x \to \infty} \frac{x}{\ln x}$

14) $\lim_{x \to 0} \frac{e^{x} - 1}{2x}$

15) $\lim_{x \to 0^{+}} \frac{5 \ln \tan x}{\ln \sin x}$

16) $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$

17) $\lim_{x \to \infty} \frac{e^{2x}}{2x^{2}}$

18) $\lim_{x \to \infty} \frac{2x^{2}}{e^{x}}$

19) $\lim_{x \to \infty} \frac{\ln x^{4}}{\ln (x+1)^{2}}$

20) $\lim_{x \to 0} \frac{2x^{2}}{e^{x} - 1 - x}$

Calculus Practice 1.1C3: Limits Using L'Hospital's Rule 1a

Evaluate each limit using L'Hôpital's Rule.

1) $\lim_{x\to\infty} \frac{\ln x}{x}$

*A) 0 B) ∞
C) −2 D) 1

2) $\lim_{x\to 0^+} \frac{4\ln\tan x}{\ln\sin x}$

A) 0 B) −2
C) ∞ *D) 4

3) $\lim_{x\to\infty} \frac{2x}{e^{2x}}$

A) −1 B) ∞
C) 2 *D) 0

4) $\lim_{x\to -1} \frac{2\left(x^2-1\right)}{\ln x^2}$

A) 0 *B) 2
C) 1 D) ∞

5) $\lim_{x\to 0} \frac{e^x-1}{5x}$

*A) $\frac{1}{5}$ B) ∞
C) 0 D) 1

6) $\lim_{x\to\infty} \frac{e^x}{2x}$

A) 1 B) 0
C) −3 *D) ∞

7) $\lim_{x\to -1} \frac{4\ln x^2}{x^2-1}$

*A) 4 B) ∞
C) 0 D) −1

8) $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$

A) 3 *B) $\frac{1}{6}$
C) ∞ D) 0

9) $\lim_{x\to 0} \frac{4\left(e^x-e^{-x}\right)}{\sin(2x)}$

A) −2 *B) 4
C) 0 D) ∞

10) $\lim_{x\to 2} \frac{\sqrt{x+14}-4}{x-2}$

A) 4 B) ∞
*C) $\frac{1}{8}$ D) 0

11) $\lim_{x \to 0} \frac{e^{5x} - 1}{\sin(5x)}$

*A) 1 B) ∞
C) 5 D) 0

12) $\lim_{x \to 1} \frac{2(x - 1)}{\ln x}$

*A) 2 B) 0
C) ∞ D) 4

13) $\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$

A) ∞ B) 0
C) 1 *D) 5

14) $\lim_{x \to 0^+} \frac{\ln \sin x}{\ln \tan x}$

*A) 1 B) ∞
C) 0 D) −1

15) $\lim_{x \to 1} \frac{2 \ln x}{x - 1}$

*A) 2 B) 0
C) ∞ D) −4

16) $\lim_{x \to 0} \frac{4x}{\ln(x + 1)}$

A) ∞ B) −4
*C) 4 D) 0

17) $\lim_{x \to \infty} \frac{e^x}{2x^2}$

A) 0 *B) ∞
C) −3 D) 1

18) $\lim_{x \to 0} \frac{2(e^x - 1 - x)}{x^2}$

A) ∞ B) 2
C) 0 *D) 1

19) $\lim_{x \to 0} \frac{x^2}{e^x - 1 - x}$

*A) 2 B) 0
C) 1 D) ∞

20) $\lim_{x \to \infty} \frac{2x^2}{e^{2x}}$

A) ∞ B) −3
C) 2 *D) 0

Calculus Practice 1.1C3: Limits Using L'Hospital's Rule 1b

Evaluate each limit using L'Hôpital's Rule.

1) $\lim_{x \to 0} \frac{\sin(3x)}{\sin(5x)}$

$\frac{3}{5}$

2) $\lim_{x \to 0} \frac{\tan(3x)}{x}$

3

3) $\lim_{x \to 0^+} \frac{\ln \sin x}{\ln \tan x}$

1

4) $\lim_{x \to 0} \frac{e^x - e^{-x}}{x}$

2

5) $\lim_{x \to 1} \frac{2 \ln x}{x - 1}$

2

6) $\lim_{x \to -1} \frac{3 \ln x^2}{x^2 - 1}$

3

7) $\lim_{x \to -3} -\frac{x^2 - x - 12}{x + 3}$

7

8) $\lim_{x \to 0} \frac{3\left(e^x - e^{-x}\right)}{\sin x}$

6

9) $\lim_{x \to \infty} \frac{\ln x}{2x}$

0

10) $\lim_{x \to 0} \frac{e^{2x} - 1}{3x^3}$

∞

11) $\lim_{x \to 0} \frac{e^{3x} - 1}{\sin(3x)}$

1

12) $\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25}$

$\frac{1}{10}$

13) $\lim_{x \to \infty} \frac{x}{\ln x}$

∞

14) $\lim_{x \to 0} \frac{e^{x} - 1}{2x}$

$\frac{1}{2}$

15) $\lim_{x \to 0^+} \frac{5 \ln \tan x}{\ln \sin x}$

5

16) $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$

$\frac{1}{4}$

17) $\lim_{x \to \infty} \frac{e^{2x}}{2x^2}$

∞

18) $\lim_{x \to \infty} \frac{2x^2}{e^{x}}$

0

19) $\lim_{x \to \infty} \frac{\ln x^4}{\ln (x+1)^2}$

2

20) $\lim_{x \to 0} \frac{2x^2}{e^{x} - 1 - x}$

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