

# PVA

To be successful you must accept all challenges that come your way. You can't just accept the ones you like. ~Mike Gafka

## Velocity for chapter 6

## AP Calculus AB

### NOTES:

**Displacement** of the particle on  $[a,b] = \int_a^b v(t) dt$

**Total distance** traveled of the particle on  $[a,b] = \int_a^b |v(t)| dt$

**Position of the particle at  $t = a$ :** initial position + displacement =  $s(0) + \int_0^a v(t) dt$

If given a position other than at  $t = 0$  just set up a definite integral of  $v(t)$  with the position you are given and the one you are looking for and use FTC and algebra to solve for the position you are looking for.

Ex. If given  $s(3) = 8$  and you want to find  $s(10)$ :

$$\int_3^{10} v(t) dt = s(10) - s(3) \quad \text{so then} \quad s(10) = s(3) + \int_3^{10} v(t) dt = 8 + \int_3^{10} v(t) dt$$

If instead you want to find  $s(1)$  still knowing that  $s(3) = 8$ :

$$\int_1^3 v(t) dt = s(3) - s(1) \quad \text{so then} \quad s(1) = s(3) - \int_1^3 v(t) dt = 8 - \int_1^3 v(t) dt$$

### Calculator Question:

A particle moves along the x-axis so that at any time  $t > 0$  its velocity is given by  $v(t) = t \ln t - t$ . At time  $t = 1$ , the position of the particle is  $x(1) = 6$ .

- When is the particle at rest?  $t \approx 2.718 = e$
- For what values of  $t$  is the particle moving to the right?  $(e, \infty)$
- Find the acceleration of the particle at time  $t = 5$ .  $\approx 1.609, v'(5)$
- When is the particle's acceleration zero?  $t = 1, v'(t) = 0$
- Find the total distance traveled by the particle from  $t = 1$  to  $t = 8$ .
- Find the displacement of the particle from  $t = 1$  to  $t = 8$ .  $21.487$
- Find the position of the particle at  $t = 5$ .
- Find the position of the particle at  $t = 0.3$ .
- For what values of  $t$  is the particle slowing down? Speeding up?

speeding up:  $(0, 1) \cup (e, \infty)$   
slowing down:  $(1, e)$

$\int v(t) dt \rightarrow$

$$g) \int_1^5 v(t) dt = x(5) - x(1)$$

$$2.118 = x(5) - 6$$

$$\boxed{x(5) = 8.118}$$

$$h) \int_{0.3}^1 v(t) dt = x(1) - x(0.3)$$

$$-.628 = 6 - x(0.3)$$

$$\boxed{x(0.3) = 6.628}$$



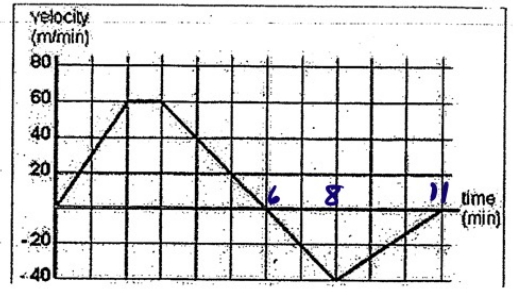
PVA

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w/ graph



1. The velocity of a particle is given on the right. The position of the particle at  $t=0$  is 3.

- What is the total distance traveled from  $t=0$  to  $t=6$ ?

$$60 \left( \frac{1+6}{2} \right) = \boxed{210 \text{ m}}$$

- What is the total distance traveled from  $t=0$  to  $t=8$ ?

$$210 + \frac{1}{2}(2)(40) = \boxed{250 \text{ m}}$$

- What is the total distance traveled from  $t=0$  to  $t=11$ ?

$$250 + \frac{1}{2}(3)(40) = \boxed{310 \text{ m}}$$

- What is the displacement of the particle from  $t=0$  to  $t=6$ ?

$$\boxed{210 \text{ m}}$$

- What is the displacement of the particle from  $t=0$  to  $t=8$ ?

$$210 - 40 = \boxed{170 \text{ m}}$$

- What is the displacement of the particle from  $t=0$  to  $t=11$ ?

$$170 - 40 = \boxed{110 \text{ m}}$$

- Where is the particle at  $t=6$ ? At  $t=11$ ?

$$\int_0^6 v(t) dt = s(6) - s(0)$$

$$210 = s(6) - 3$$

$$\boxed{s(6) = 213}$$

$$\int_0^{11} v(t) dt = s(11) - s(0)$$

$$110 = s(11) - 3$$

$$\boxed{s(11) = 113}$$

Study the FTOC!



w/o calc

2. Given  $v(t) = 4 - t^2$  and  $s(0) = -4$  units

- Find the total distance the particle traveled from  $t=0$  to 3 seconds.

$$\int v(t) dt = \int (4 - t^2) dt = 4t - \frac{1}{3}t^3 + C$$

$$-4 = C \quad s(t) = -\frac{1}{3}t^3 + 4t - 4$$

- Find the displacement of the particle from  $t=0$  to 3 seconds.

$$\int_0^3 (4 - t^2) dt = 4t - \frac{1}{3}t^3 \Big|_0^3 = 4(3) - \frac{1}{3}(3)^3 = 12 - 9 = \boxed{3}$$

x	s(t)
0	-4
2	$1\frac{1}{3}$
3	-1

$5\frac{1}{3}$  right  
 $2\frac{1}{3}$  left  
 $\boxed{7\frac{2}{3}}$

- Where is the particle at  $t=3$  sec?

$$\int_0^3 v(t) dt = s(3) - s(0)$$

$$3 = s(3) - (-4)$$

$$3 = s(3) + 4$$

$$\boxed{s(3) = -1}$$

or

$$s(3) = -\frac{1}{3}(3)^3 + 4(3) - 4$$

$$-\frac{1}{3}(27) + 12 - 4$$

$$-9 + 12 - 4$$

$$3 - 4 = \boxed{-1}$$

