How to justify in AP Calculus AB

Stuff to remember:

Critical values/numbers are where $f'(x)$ is either equal to zero OR $f'(x)$ is undefined [but $f(x)$ is defined]

Possible points of inflection are where $f''(x)$ is either equal to zero OR $f''(x)$ is undefined [but $f(x)$ is defined]

If a function is differentiable at a point, then the function is continuous at that point.

How to justify: PLEASE LOOK AT THE INTERVAL NOTATION

[Assume that the function is continuous and differentiable]

If $f'(x) > 0$ for every x in (a, b), then $f(x)$ is increasing on $[a, b]$

If $f'(x) < 0$ for every x in (a, b), then $f(x)$ is increasing on $[a, b]$

If $f''(x) > 0$ for every x in (a, b), then $f(x)$ is concave up on (a, b) AND $f'(x)$ is increasing on $[a, b]$

If $f''(x) < 0$ for every x in (a, b), then $f(x)$ is concave down on (a, b)AND $f'(x)$ is decreasing on $[a, b]$

If $f'(x)$ is increasing on (a, b), then $f(x)$ is concave up on (a, b) AND $f''(x) > 0$ on (a, b)

If $f'(x)$ is decreasing on (a, b), then $f(x)$ is concave down on (a, b) AND $f''(x) < 0$ on (a, b)

Let $x = a$ be a critical value for $f(x)$ [in other words, $f'(a) = 0$ OR $f'(a)$ is undefined but $f(a)$ is defined]

To show that $(a, f(a))$ is a relative/local minimum:
"At $x=a$, $f'(a)$ changes from negative to positive values. Hence, f has a relative minimum at $x = a$."

To show that $(a, f(a))$ is a relative/local maximum:
"At $x = a$, $f'(a)$ changes from positive to negative values. Hence, f has a relative maximum at $x = a$."

Let $f''(b) = 0$ or $f''(b)$ be undefined but $f(b)$ is defined
To show that $(b, f(b))$ is a point of inflection you must justify with one of the following statements:
"At $x = b$, $f''(b)$ changes from positive to negative values. Hence, f has a point of inflection at $x = b$."
OR
"At $x = b$, $f''(b)$ changes from negative to positive values. Hence, f has a point of inflection at $x = b$."

If given the graph of the first derivative, then a point of inflection will occur at $x = a$ if at $x=a$ the graph of $f'(x)$ changes from either increasing to decreasing OR decreasing to increasing.

NOTE: When you justify, you must be SPECIFIC. Do NOT state a rule but rather, explain what is occurring at some specific value of $x = a$.

To justify an absolute maximum or absolute minimum
The candidates are the critical values of f(x) AND the endpoints!
First justify any relative min/max AND find their function values.
Then compare the function values to the endpoint values and decide which is the absolute min/max.

Extreme Value Theorem
If a function, f, is continuous on [a, b], then f has both a maximum and a minimum value on [a, b].

Mean Value Theorem
Your justification statement should look like:
By the Mean Value Theorem there is a c, $a < c < b$, such that

$$\frac{f(b)-f(a)}{b-a}=f'(c)$$

You need to supply all of the values!

You never really need Rolle's Theorem since you can always use the Mean Value Theorem

Rolle's Theorem [A "special" case of MVT]
Your justification statement should look like:
By Rolle's Theorem, since f is continuous on [a, b] and differentiable on (a, b) AND f(a)=f(b), then there is a c, a < c < b, such that f'(c) = 0.
Note: You can always just use MVT.

Horizontal TANGENTS occur at x = a if f'(a) 0= 0
Vertical TANGENTS occur at x = a if f'(a) is undefined but f(a) is defined

If f'(x) is of the form $f'(x)=\frac{g(x)}{h(x)}$, then f will have a horizontal tangent when g(x) = 0 AND f will have a vertical tangent if h(x) = 0. [Of course f(x) must be defined for those values]

Horizontal Asymptote

y = b is a horizontal asymptote of the graph of y = f(x) if either $\lim_{x \to \infty} f(x) = b$ OR $\lim_{x \to -\infty} f(x) = b$

Limits of Rational Functions as $x \to \pm\infty$ [End Behavior]

$\lim_{x \to \pm\infty} \frac{f(x)}{g(x)} = 0$ if the degree of f(x) < the degree of g(x)

$\lim_{x \to \pm\infty} \frac{f(x)}{g(x)} = \infty$ or "dne" if the degree of f(x) > the degree of g(x)

$\lim_{x \to \pm\infty} \frac{f(x)}{g(x)}$ is finite [there is a horizontal asymptote] if

degree of f(x) = degree of g(x)

Remember: A graph is NOT justification. You must use Calculus to justify any extrema, points of inflection, horizontal asymptotes, etc.

Remember:
If $s(t)$ is the position function, then
Velocity, $v(t) = s'(t)$
Acceleration, $a(t) = v'(t) = s''(t)$
Speed is $|v(t)|$
The speed of a particle is increasing at $x = c$ if $v(c)$ and $a(c)$ are either both positive or both negative.
The speed of a particle is decreasing at $x = c$ if $v(c)$ and $a(c)$ have different signs.
To justify that the speed of a particle is increasing at $x = c$ you need to clearly state that $v(c) > 0$ AND $a(c) > 0$
OR $v(c) < 0$ AND $a(c) < 0$ in a statement.
To justify that the speed of a particle is decreasing at $x = c$ you need to clearly state that $v(c) > 0$ AND $a(c) < 0$
OR $v(c) < 0$ AND $a(c) > 0$ in a statement.

OPTIMIZATION
If necessary, you might want to draw a picture of what the problem is about.
Figure out what you are trying to optimize.
Find a primary equation that fits what you are trying to optimize.
If necessary, find a secondary equation that will allow you to re-write your primary equation in terms of just ONE VARIABLE.
Now just find the min or max using standard Calculus min/max techniques.