Topic: Euler's Method and Logistic Growth Functions
Student Saturday Session

NATIONAL MATH + SCIENCE INITIATIVE

I. Euler's Method -- Multiple Choice Examples

Example 1 2003 BC5

5. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5 ?

(A) 3 (B) 5 (C) 6 (D) 10 (E) 12

Example 2 2008 BC7

7. Given that $y(1) = -3$ and $\frac{dy}{dx} = 2x + y$, what is the approximation for $y(2)$ if Euler's method is used with a step size of 0.5, starting at $x = 1$?

(A) -5 (B) -4.25 (C) -4 (D) -3.75 (E) -3.5

2008

7. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x - y - 1$ with the initial condition $f(1) = -2$. What is the approximation for $f(1.4)$ if Euler's method is used, starting at $x = 1$ with two steps of equal size?

(A) -2 (B) -1.24 (C) -1.2 (D) -0.64 (E) 0.2

2013

9. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 2x + y$ with initial condition $f(1) = 0$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length, starting at $x = 1$?

(A) 0 (B) 1 (C) 2.75 (D) 3 (E) 6

2014

x	$f'(x)$
1	0.2
1.5	0.5
2	0.9

83. The table above gives values of f', the derivative of a function f. If $f(1) = 4$, what is the approximation to $f(2)$ obtained by using Euler's method with a step size of 0.5 ?

(A) 2.35
(B) 3.65
(C) 4.35
(D) 4.70
(E) 4.80

2015

9. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 2y - x$ with initial condition $f(1) = 2$. What is the approximation for $f(0)$ obtained by using Euler's method with two steps of equal length starting at $x = 1$?

(A) $-\frac{5}{4}$ (B) -1 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) $\frac{27}{4}$

2016

x	-0.2	0	0.2	0.4
$f'(x)$	0.8	1.2	1.7	2.3

77. The table above shows values of f', the derivative of a function f, for selected values of x. If $f(-0.2) = 1$, what is the approximation for $f(0.4)$ obtained by using Euler's method with a step size of 0.2 starting at $x = -0.2$?

(A) 1.48 (B) 1.74 (C) 2.04 (D) 2.20

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Question 5

Let f be the function satisfying $f'(x) = -3xf(x)$, for all real numbers x, with $f(1) = 4$ and $\lim_{x\to\infty} f(x) = 0$.

(a) Evaluate $\int_1^\infty -3xf(x)\,dx$. Show the work that leads to your answer.

(b) Use Euler's method, starting at $x = 1$ with a step size of 0.5, to approximate $f(2)$.

(c) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = -3xy$ with the initial condition $f(1) = 4$.

Question 5

Consider the differential equation $\frac{dy}{dx} = 3x + 2y + 1.$

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

(b) Find the values of the constants m, b, and r for which $y = mx + b + e^{rx}$ is a solution to the differential equation.

(c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = -2$. Use Euler's method, starting at $x = 0$ with a step size of $\frac{1}{2}$, to approximate $f(1)$. Show the work that leads to your answer.

(d) Let $y = g(x)$ be another solution to the differential equation with the initial condition $g(0) = k$, where k is a constant. Euler's method, starting at $x = 0$ with a step size of 1, gives the approximation $g(1) \approx 0$. Find the value of k.

2008 BC5

5. Let f be the function satisfying $f'(x) = 4x - 2xf(x)$ for all real numbers x, with $f(0) = 5$ and $\lim_{x \to \infty} f(x) = 2$.

(a) Find the value of $\int_0^{\infty} (4x - 2xf(x))\, dx$. Show the work that leads to your answer.

(b) Use Euler's method to approximate $f(-1)$, starting at $x = 0$, with two steps of equal size.

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 4x - 2xy$ with the initial condition $f(0) = 5$.

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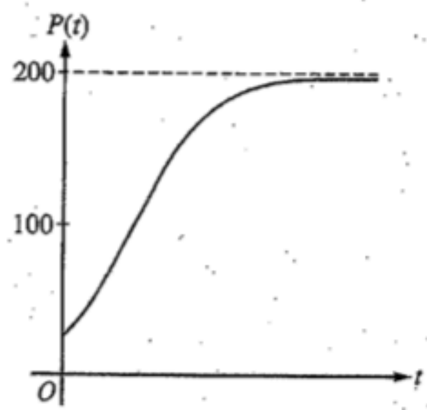
III. Logistic Growth Functions – Multiple Choice

Example 1 2003 BC21

21. The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{dM}{dt} = 0.6M\left(1 - \frac{M}{200}\right)$, where t is the time in years and $M(0) = 50$. What is $\lim_{t\to\infty} M(t)$?

(A) 50 (B) 200 (C) 500 (D) 1000 (E) 2000

Example 2 2008 BC 24



24. Which of the following differential equations for a population P could model the logistic growth shown in the figure above?

(A) $\frac{dP}{dt} = 0.2P - 0.001P^2$

(B) $\frac{dP}{dt} = 0.1P - 0.001P^2$

(C) $\frac{dP}{dt} = 0.2P^2 - 0.001P$

(D) $\frac{dP}{dt} = 0.1P^2 - 0.001P$

(E) $\frac{dP}{dt} = 0.1P^2 + 0.001P$

2008

84. The rate of change, $\frac{dP}{dt}$, of the number of people on an ocean beach is modeled by a logistic differential equation. The maximum number of people allowed on the beach is 1200. At 10 A.M., the number of people on the beach is 200 and is increasing at the rate of 400 people per hour. Which of the following differential equations describes the situation?

(A) $\frac{dP}{dt} = \frac{1}{400}(1200 - P) + 200$

(B) $\frac{dP}{dt} = \frac{2}{5}(1200 - P)$

(C) $\frac{dP}{dt} = \frac{1}{500}P(1200 - P)$

(D) $\frac{dP}{dt} = \frac{1}{400}P(1200 - P)$

(E) $\frac{dP}{dt} = 400P(1200 - P)$

2013

7. A population y changes at a rate modeled by the differential equation $\frac{dy}{dt} = 0.2y(1000 - y)$, where t is measured in years. What are all values of y for which the population is increasing at a decreasing rate?

(A) 500 only

(B) $0 < y < 500$ only

(C) $500 < y < 1000$ only

(D) $0 < y < 1000$

(E) $y > 1000$

2014

77. The number of antibodies y in a patient's bloodstream at time t is increasing according to a logistic differential equation. Which of the following could be the differential equation?

(A) $\frac{dy}{dt} = 0.025t$

(B) $\frac{dy}{dt} = 0.025t(5000 - t)$

(C) $\frac{dy}{dt} = 0.025y$

(D) $\frac{dy}{dt} = 0.025(5000 - y)$

(E) $\frac{dy}{dt} = 0.025y(5000 - y)$

2015

13. A population of wolves is modeled by the function P and grows according to the logistic differential equation $\frac{dP}{dt} = 5P\left(1 - \frac{P}{5000}\right)$, where t is the time in years and $P(0) = 1000$. Which of the following statements are true?

I. $\lim_{t \to \infty} P(t) = 5000$

II. $\frac{dP}{dt}$ is positive for $t > 0$.

III. $\frac{d^2P}{dt^2}$ is positive for $t > 0$.

(A) I only

(B) II only

(C) I and II only

(D) I and III only

(E) I, II, and III

2016

28. The function N satisfies the logistic differential equation $\frac{dN}{dt} = \frac{N}{10}\left(1 - \frac{N}{850}\right)$, where $N(0) = 105$. Which of the following statements is false?

(A) $\lim_{t \to \infty} N(t) = 850$

(B) $\frac{dN}{dt}$ has a maximum value when $N = 105$.

(C) $\frac{d^2N}{dt^2} = 0$ when $N = 425$.

(D) When $N > 425$, $\frac{dN}{dt} > 0$ and $\frac{d^2N}{dt^2} < 0$.

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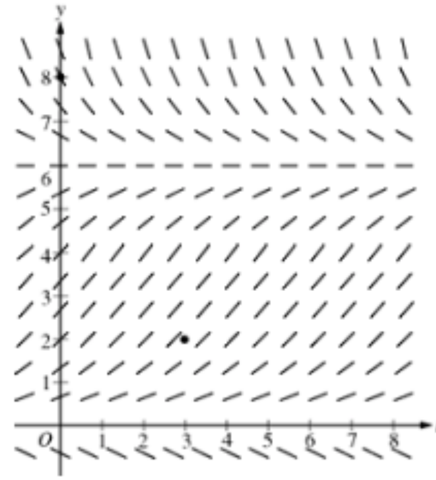
Question 6

Consider the logistic differential equation $\frac{dy}{dt} = \frac{y}{8}(6 - y)$. Let $y = f(t)$ be the particular solution to the differential equation with $f(0) = 8$.

(a) A slope field for this differential equation is given below. Sketch possible solution curves through the points $(3, 2)$ and $(0, 8)$.



(b) Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$.

(c) Write the second-degree Taylor polynomial for f about $t = 0$, and use it to approximate $f(1)$.

(d) What is the range of f for $t \geq 0$?

Question 5

A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right).$$

(a) If $P(0) = 3$, what is $\lim_{t\to\infty} P(t)$?

If $P(0) = 20$, what is $\lim_{t\to\infty} P(t)$?

(b) If $P(0) = 3$, for what value of P is the population growing the fastest?

(c) A different population is modeled by a function Y that satisfies the separable differential equation

$$\frac{dY}{dt} = \frac{Y}{5}\left(1 - \frac{t}{12}\right).$$

Find $Y(t)$ if $Y(0) = 3$.

(d) For the function Y found in part (c), what is $\lim_{t\to\infty} Y(t)$?

2015 BC4

4. In a national park, the population of mountain lions grows over time. At time $t = 0$, where t is measured in years, the population is found to be 20 mountain lions.

   (a) One zoologist suggests a population model P that satisfies the differential equation $\frac{dP}{dt} = \frac{1}{4}(220 - P)$. Use separation of variables to solve this differential equation for P with the initial condition $P(0) = 20$.

(b) A second zoologist suggests a population model Q that satisfies $\frac{dQ}{dt} = \frac{1}{500}Q(220 - Q)$. Find the value of $\frac{dQ}{dt}$ at the time when Q grows most rapidly.

(c) For the population model Q introduced in part (b), use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $Q(10)$. Show the computations that lead to your answer.