Name ____ Date __________

Chapter 9 (Sequences and Series)
AP Practice Problems

Part 1: No Calculator

1.

. What is the value of $\sum_{n=0}^{\infty}\left(-\frac{2}{3}\right)^n$?

(A) -2 (B) $-\frac{2}{5}$ (C) $\frac{3}{5}$ (D) 3 (E) The series diverges.

2.

. Which of the following series converge?

I. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$

II. $\sum_{n=1}^{\infty}\frac{3^n}{n!}$

III. $\sum_{n=1}^{\infty}\left(\frac{e}{\pi}\right)^n$

(A) None
(B) II only
(C) III only
(D) I and II only
(E) II and III only

3.

. What is the value of $\sum_{n=1}^{\infty}\frac{(-3)^{n+1}}{5^n}$?

(A) $-\frac{15}{8}$ (B) $-\frac{9}{8}$ (C) $-\frac{3}{8}$ (D) $\frac{9}{8}$ (E) $\frac{15}{8}$

4.

Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{|\sin n|}{n^2}$ II. $\sum_{n=1}^{\infty} e^{-n}$ III. $\sum_{n=1}^{\infty} \frac{n+2}{n^2+n}$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

5.

The infinite series $\sum_{k=1}^{\infty} a_k$ has nth partial sum $S_n = (-1)^{n+1}$ for $n \geq 1$. What is the sum of the series $\sum_{k=1}^{\infty} a_k$?

(A) -1

(B) 0

(C) $\frac{1}{2}$

(D) 1

(E) The series diverges.

6.

). What is the sum of the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}}$?

(A) $\frac{-2}{e^2 - 2e}$ (B) $\frac{-2}{e^2 + 2e}$ (C) $\frac{-2}{e+2}$ (D) $\frac{e}{e+2}$ (E) The series diverges.

7.

ł. Which of the following series converge?

I. $1 + (-1) + 1 + \cdots + (-1)^{n-1} + \cdots$

II. $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1} + \cdots$

III. $1 + \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^{n-1}} + \cdots$

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

8.

. Which of the following series converges?

(A) $\sum_{n=1}^{\infty} \frac{3n}{n+2}$

(B) $\sum_{n=1}^{\infty} \frac{3n}{n^2+2}$

(C) $\sum_{n=1}^{\infty} \frac{3n}{n^2+2n}$

(D) $\sum_{n=1}^{\infty} \frac{3n^2}{n^3+2n}$

(E) $\sum_{n=1}^{\infty} \frac{3n^2}{n^4+2n}$

9.

Consider the geometric series $\sum_{n=1}^{\infty} a_n$, where $a_n > 0$ for all n. The first term of the series is $a_1 = 48$, and the third term is $a_3 = 12$. Which of the following statements about $\sum_{n=1}^{\infty} a_n$ is true?

(A) $\sum_{n=1}^{\infty} a_n = 64$

(B) $\sum_{n=1}^{\infty} a_n = 96$

(C) $\sum_{n=1}^{\infty} a_n$ converges, but the sum cannot be determined from the information given.

(D) $\sum_{n=1}^{\infty} a_n$ diverges.

10.

Which of the following series can be used with the limit comparison test to determine whether the series $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ converges or diverges?

(A) $\sum_{n=1}^{\infty} \frac{1}{n}$ (B) $\sum_{n=1}^{\infty} \frac{1}{n^3}$ (C) $\sum_{n=1}^{\infty} \frac{n}{n+1}$ (D) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

11.

Which of the following series are conditionally convergent?

I. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$

III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

Part 2: Calculator

12.

. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \geq 1$. If $\sum_{n=1}^{\infty} a_n$ converges, which of the following must be true?

(A) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(B) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(C) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(D) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(E) If $b_n \leq a_n$, then the behavior of $\sum_{n=1}^{\infty} b_n$ cannot be determined from the information given.

13.

. Which of the following statements are true about the series $\sum_{n=2}^{\infty} a_n$, where $a_n = \dfrac{(-1)^n}{\sqrt{n} + (-1)^n}$?

I. The series is alternating.

II. $|a_{n+1}| \leq |a_n|$ for all $n \geq 2$

III. $\lim_{n\to\infty} a_n = 0$

(A) None

(B) I only

(C) I and II only

(D) I and III only

(E) I, II, and III

14.

. If $0 < b_n < a_n$ for $n \geq 1$, which of the following must be true?

(A) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} b_n$ converges.

(B) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} b_n = 0$.

(C) If $\sum_{n=1}^{\infty} b_n$ diverges, then $\lim_{n\to\infty} a_n = 0$.

(D) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

(E) If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

15.

The infinite series $\sum_{k=1}^{\infty} a_k$ has nth partial sum $S_n = \frac{n}{3n+1}$ for $n \geq 1$. What is the sum of the series $\sum_{k=1}^{\infty} a_k$?

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$ (E) The series diverges.

16.

. The alternating series test can be used to show convergence of which of the following alternating series?

I. $4-\frac{1}{9}+1-\frac{1}{81}+\frac{1}{4}-\frac{1}{729}+\frac{1}{16}-\cdots+a_n+\cdots$, where $a_n=\begin{cases}\frac{8}{2^n} & \text{if } n \text{ is odd}\\ -\frac{1}{3^n} & \text{if } n \text{ is even}\end{cases}$

II. $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}-\frac{1}{8}+\cdots+a_n+\cdots$, where $a_n=\frac{(-1)^{n+1}}{n}$

III. $\frac{2}{3}-\frac{3}{5}+\frac{4}{7}-\frac{5}{9}+\frac{6}{11}-\frac{7}{13}+\frac{8}{15}-\cdots+a_n+\cdots$, where $a_n=(-1)^{n+1}\frac{n+1}{2n+1}$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III