AP Calculus

Name: _____

Block: _____

Chapter 7 Test Review

For questions 1 – 21, integrate each of the following indefinite integrals.

1. $\int \frac{3x}{\sqrt[3]{x^2+3}}\,dx$

2. $\int \sec(5x)\,dx$

3. $\int \frac{x-1}{x+1}\,dx$

4. $\int 9\sin x\,dx$

5. $\int 7^{3x}\,dx$

6. $\int 2\tan(x)\,dx$

7. $\int \cot^2 x\,dx$

8. $\int \frac{(x+1)^2}{x^{1/3}}\,dx$

9. $\int \cos^2(8x)\,dx$

10. $\int \frac{dx}{4+9x^2}$

11. $\int \left(10\cos t + \sin^2(10t)\right)dt$

12. $\int \sec^2 x\,dx$

13. $\int \frac{dx}{x\sqrt{x}-1}$

14. $\int 5e^{3x}\cot\left(e^{3x}\right)dx$

15. $\int \frac{1}{\sec(12x)}\,dx$

16. [illegible]

17. $\int \tan^2\left(\frac{x}{5}\right) dx$

18. $\int \frac{x}{x^2-4} dx$

19. $\int \frac{e^x}{\sqrt{1-e^{2x}}} dx$

20. $\int \csc^2 x \, dx$

21. $\int xe^{-x^2+3} \, dx$

For questions 22 – 26, evaluate each definite integral without a calculator. Check your answer with your calculator.

22. $\int_{-1}^{1} \left(x^2-5\right)^2 dx$

23. $\int_{-4}^{-2} \frac{dx}{x^2+6x+10}$

24. $\int_{0}^{1} \frac{3+x}{x^2+1} dx$

25. $\int_{0}^{6} \frac{dx}{7-x}$

26. $\int_{4}^{9} \sqrt{x}\left(3-4x\right) dx$

27. Derive (SHOW EVERY STEP) $y = y_0 e^{kt}$ from $\frac{dy}{dt} = ky$ and $y(0) = y_0$.

28. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2 + 1}{2y}$.

a) Find the slope of the graph of f at the point where $x = 1$.

b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$

c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2 + 1}{2y}$ with the initial condition $f(1) = 4$.

d) Use your solution from part *c* to find $f(1.2)$

29. If $\frac{dy}{dx} = y \sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

A $e^{\tan x} + 4$
B $e^{\tan x} + 5$
C $5e^{\tan x}$
D $\tan x + 5$
E $\tan x + 5e^x$

30. [No Calculator] If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

A) $-\frac{1}{2}\ln 2$

B) $-\frac{1}{4}$

C) $\frac{1}{2}\ln 2$

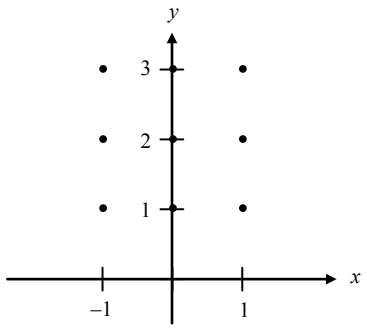
D) $\frac{\sqrt{2}}{2}$

E) $\ln 2$

31. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.

b) Draw a particular solution if $f(0) = 3$



c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$. Use your solution to find $f(0.2)$.

32. [Calculator] During a certain epidemic, the number of people that are infected at any time increases at rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

33. [Calculator] Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

34. [No Calculator] Bacteria in a certain culture increase at rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

35. Solutions to the differential equation $\frac{dy}{dx} = xy^3$ will also satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

b) Use the tangent line equation from part *a* to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.