Name: _____ **Period:** _____

Date: _____ **AP Calc BC**

Mr. Mellina

Chapter 7 Review: Applications of Integration

Topics:

1. *Area Between Curves*
2. *Volumes by Cross Section*
3. *Volumes by Disk & Washer Method*
4. *Arc Length*

HW Sets

Topic 1: Chapter 7 Review Set A
Topics 2 & 3: Chapter 7 Review Set B
Topic 4: Chapter 7 Review Set C

Topic 1: Area Between Two Curves (Day 1)

These problems were selected from the Review Exercises on pages 511 & 512.

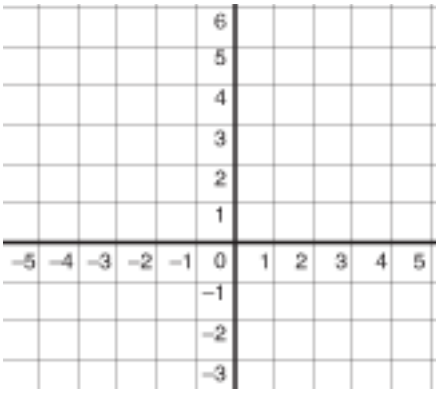
AREA OF A REGION BETWEEN TWO CURVES

If f and g are continuous on $[a, b]$ and $g(x) \le f(x)$ for all x in $[a, b]$, then the area of the region bounded by the graphs of f and g and the vertical lines $x = a$ and $x = b$ is

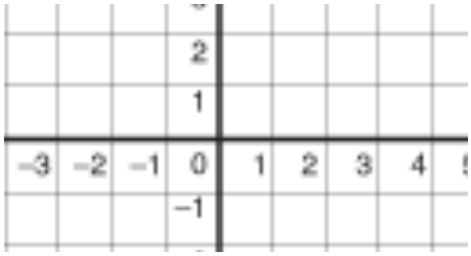
$$A = \int_a^b [f(x) - g(x)]\,dx.$$

In exercises 1-9, sketch the region bounded by the graphs of the equations and find the area of the region.

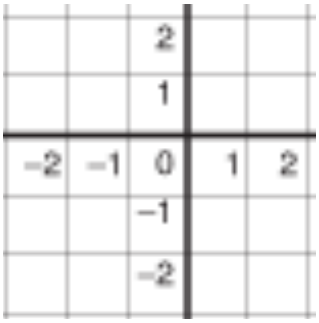
1. $y = 6 - \frac{1}{2}x^2,\ y = \frac{3}{4}x,\ x = -2,\ x = 2$



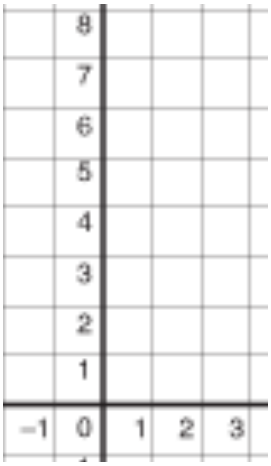
3. $y = \frac{1}{x^2+1},\ y = 0,\ x = -1,\ x = 1$



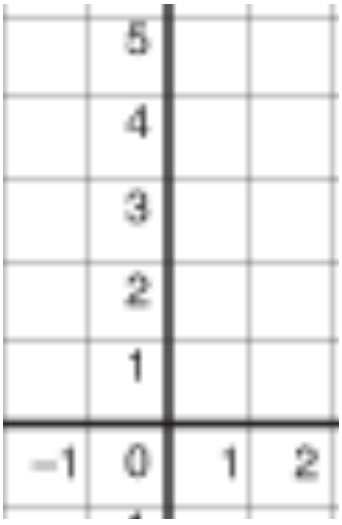
5. $y = x,\ y = x^3$



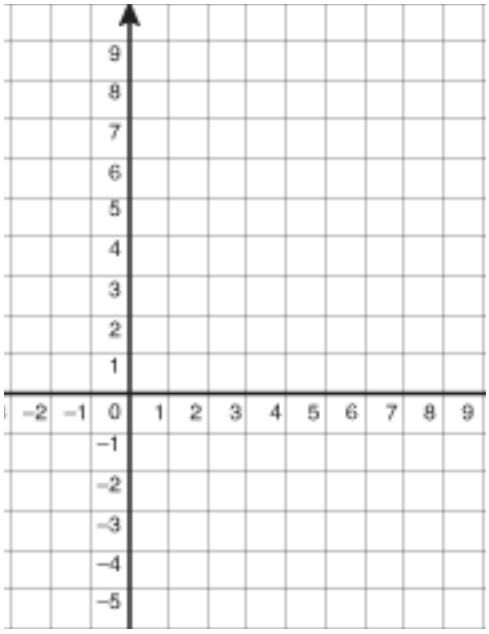
7. $y = e^x,\ y = e^2,\ x = 0$



10. $x = \cos y\,,\ x = \frac{1}{2}\,,\ \frac{\pi}{3} \le y \le \frac{7\pi}{3}$



11. $y = x^2 - 8x + 3,\ y = 3 + 8x - x^2$



Topic 2: Volumes by Cross Section (Day 2)

These problems were selected from the Exercises on page 464.

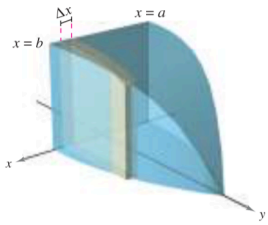
VOLUMES OF SOLIDS WITH KNOWN CROSS SECTIONS

1. For cross sections of area $A(x)$ taken perpendicular to the x-axis,

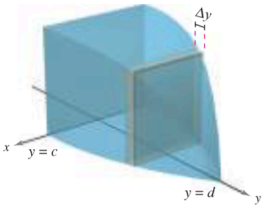
$$\text{Volume} = \int_a^b A(x)\,dx.$$ See Figure 7.24(a).

2. For cross sections of area $A(y)$ taken perpendicular to the y-axis,

$$\text{Volume} = \int_c^d A(y)\,dy.$$ See Figure 7.24(b).



(a) Cross sections perpendicular to x-axis



(b) Cross sections perpendicular to y-axis

Figure 7.24

Click here for Geogebra activity if you need help visualizing.

73. Find the volumes of the solids whose bases are bounded by the graphs of $y = x + 1$ and $y = x^2 - 1$, with the indicated cross sections taken perpendicular to the x-axis.

    a. Squares

    b. Rectangles of height 1

74. Find the volumes of the solids whose bases are bounded by the circle $x^2 + y^2 = 4$, with the indicated cross sections taken perpendicular to the x-axis.

a. Squares

b. Equilateral Triangles

c. Semicircles

d. Isosceles Right Triangles

Topic 3: Volumes using the Disk & Shell Methods (Day 3)

These problems were selected from the Review Exercises on pages 511 & 512.

THE DISK METHOD

To find the volume of a solid of revolution with the **disk method,** use one of the following, as shown in Figure 7.15.

Horizontal Axis of Revolution	*Vertical Axis of Revolution*
Volume $= V = \pi \int_a^b [R(x)]^2 \, dx$	Volume $= V = \pi \int_c^d [R(y)]^2 \, dy$

The Washer Method

$$V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) \, dx.$$

For exercise 19, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

19. $y = \frac{1}{\sqrt{1+x^2}}, y = 0, \ x = -1, \ x = 1$

For exercise 21, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the y-axis.

21. $y = \frac{1}{x^4+1}, y = 0, \ x = 0, \ x = 1$

For exercise 23, find the volumes of the solids generated by revolving the region bounded by the graphs of the equations about the given lines.

23. $y = x,\ y = 0,\ x = 3$

a. the x-axis

b. the y-axis

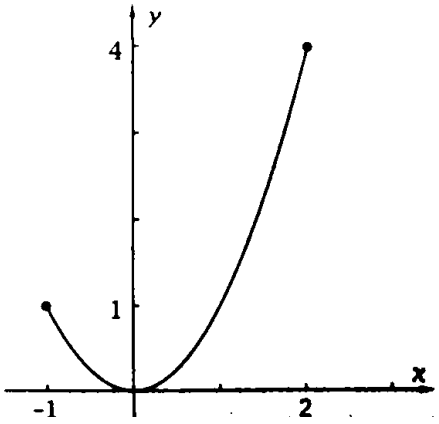
c. the line x = 3

d. the line x = 6

Topic 4: Arc Length (Day 4)

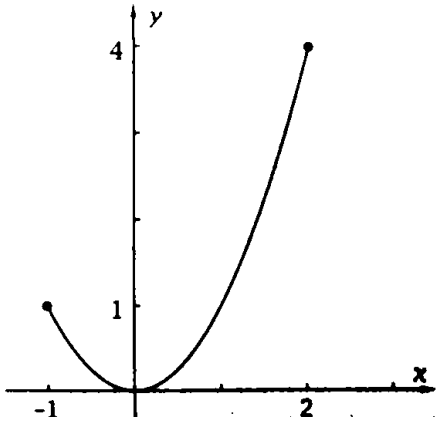
Warm Up:

a. Draw line segments connecting (consecutively) the points on the graph where *x* is -1, 0, 1, and 2. Estimate the length of these segments.



b. Does this overestimate or underestimate the arc length?

c. Find an estimate of the length using segments connecting the points on the graph where *x* is -1, -0.5, 0, 0.5, 1, 1.5, and 2.



d. Explain why this estimate is better than the one in part (a).

e. What can we do to get the exact value of the arc length?

Length of one Segment of the Arc Length = $\sqrt{(dx)^2 + (dy)^2}$

With Respect to *x*

With Respect to *y*

With Respect to *t*

Arc Length

Let the function $y = f(x)$ *represent a smooth curve on the interval* $[a, b]$. *The arc length of f between a and b is*

$s =$

Similarly, for a smooth curve $x = g(y)$, *the arc length of g between c and d is*

$s =$

Similarly, if a smooth curve C is given by $x = f(t)$ *and* $y = g(t)$ *such that C does not intersect itself on the interval* $a \leq t \leq b$ *(except possibly at the endpoints), then the length of C over the interval is given by*

$s =$

These problems were selected from the Review Exercises on pages 511 & 512.

In exercises 27 and 28, find the arc length of the graph of the function over the indicated interval.

27. $f(x) = \frac{4}{5}x^{5/4},\ [0, 4]$

28. $y = \frac{1}{3}x^{3/2} - 1,\ [2, 6]$

These problems were selected from the Exercises on page 716.
In exercises 49-53, find the arc length of the curve on the given interval.

49. $x = 3t + 5,\ y = 7 - 2t,\quad -1 \leq t \leq 3$

51. $x = e^{-t}\cos t\,,\ y = e^{-t}\sin t\,,\ \ 0 \le t \le \frac{\pi}{2}$

53. $x = \sqrt{t},\ y = 3t - 1,\ \ 0 \le t \le 1$