

Topics on the test – Sec. 7.1, 7.2, 7.4: Integration with u-sub, differential equations, exponential growth and decay, and slope fields

Evaluate each integral:

1. $\int \frac{-x}{\sqrt{4-x^2}} dx$

2. $\int \frac{5}{1-x} dx$

3. $\int \sin(6x)e^{\cos(6x)} dx$

4. $\int \frac{3x^4 - 6x^2 + 2x - 3}{x^2} dx$

5. $\int_0^{\frac{\pi}{4}} \tan x dx$

6. $\int_1^e \frac{\ln x}{x} dx$

7. $\int \frac{x^2}{(16-x^3)^2} dx$

8. $\int \tan^4 x \sec^2 x dx$

9. $\int 2x(x^2 - 1)^4 dx$

10. $\int \frac{dx}{(x+2)^3}$

11. $\int_0^1 xe^{x^2} dx$

12. $\int_0^{\pi/6} \sin(3x) dx$

13. Write and solve a differential equation that models the verbal statement:

a) The rate of change of P with respect to t is proportional to $10 - t$.

b) N is changing at a rate proportional to N. When $t = 0$, $N = 250$ and when $t = 1$, $N = 400$. What is the value of N when $t = 4$?

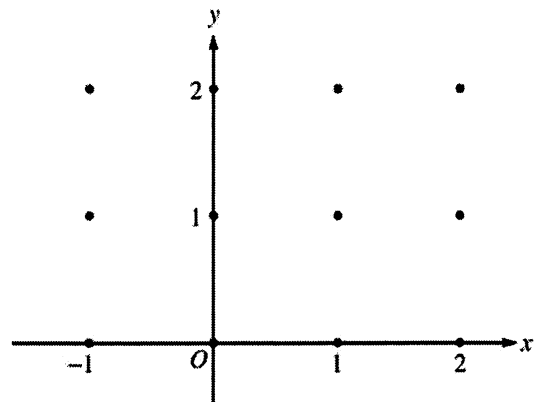
14. Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

(Note: Use the axes provided in the test booklet.)

(b) Write an equation for the line tangent to the graph of f at $x = -1$.

(c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.



15.

Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

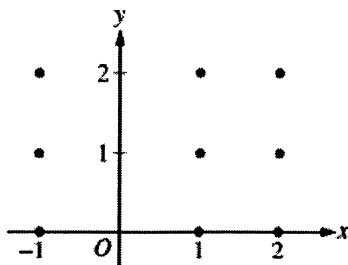
- Use separation of variables to find an expression for v in terms of t , where t is measured in seconds.
- Terminal velocity is defined as $\lim_{t \rightarrow \infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.
- It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

16.

2008 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

5. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

- On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)



- Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.
- For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \rightarrow \infty} f(x)$.