Name: ______________________

Block: ____

Chapter 6 Test Review

1. [No Calculator] Evaluate using the FTOC (the evaluation part)

a) $\int_{2}^{7}\left(\frac{8}{x}+7\sqrt[3]{x}+x^{-4}\right)dx$

b) $\int_{4}^{9}\left(\frac{5}{x^3}+7\sqrt{x}+\frac{1}{x}\right)dx$

2. [No Calculator] Evaluate using geometry

a) $\int_{-2}^{3}\sqrt{25-(x+2)^2}\,dx$

c) $\int_{-6}^{1}|8+2x|\,dx$

3. [No Calculator] Evaluate each derivative.

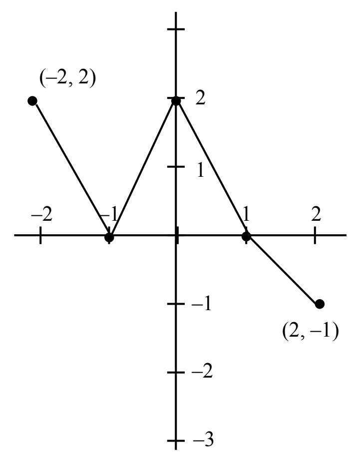
a) $\frac{d}{dx}\left[\int_{10}^{x}\tan\left(3t^2+9\right)dt\right]$

b) Find $h'(x)$ if $h(x)=\int_{5x^4}^{\sec x}\sqrt{4t-9}\,dt$.

c) $\frac{d}{dx}\left[\int_{8}^{x}\ln\left(3t^2+9\right)dt\right]$

d) Find $h'(x)$ if $h(x)=\int_{3x^6}^{\tan x}\frac{9}{t^2-1}\,dt$.

4. [No Calculator] Given the graph of $f(x)$ as shown and the definition of $g(x)=\int_{0}^{x}f(t)\,dt$



Graph of f

a) Find $g(-1)$, $g'(-1)$, $g''(-1)$

b) Over what interval is $g(x)$ increasing.
Show your work and explain your reasoning.

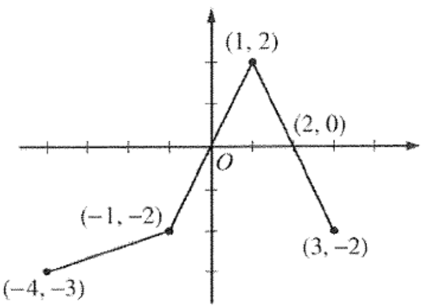
c) Over what interval is $g(x)$ concave up? Show your work and explain your reasoning.

d) Graph $g(x)$

5. [No Calculator] The graph of the function f shown below consists of three line segments.

a) Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$.

For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.



Graph of f

b) For the function g defined in part a, find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

c) Let h be the function given by $h(x) = \int_{x}^{3} f(t)\,dt$.

Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

d) For the function h defined in part c, find all intervals on which h is decreasing. Explain your reasoning.

6. [Calculator] The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table below shows the water temperature as recorded every 3 days over a 15-day period.

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

d) Use the function P defined in part c to find the average value, in °C, of $P(t)$ over the time interval $0 \le t \le 15$ days.

7. [Calculator] For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

a) Show that the number of mosquitoes is increasing at time $t = 6$.

b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

8. [No Calculator] Suppose $\int_1^2 f(x)\,dx = 3$, $\int_1^5 f(x)\,dx = -13$, and $\int_1^5 g(x)\,dx = 7$. Find each of the following:

a) $\int_3^3 g(x)\,dx$

b) $\int_5^1 f(x)\,dx$

c) $\int_1^5 \left[g(x) - f(x)\right]dx$

d) $\int_2^5 f(x)$

e) $\int_1^5 \left[3f(x) - g(x)\right]dx$

f) $\int_1^5 \frac{g(x)}{4}\,dx$

9. [No Calculator] Suppose $H(x)=\int_2^x \ln(t+5)\,dt$ for the interval [2, 10].

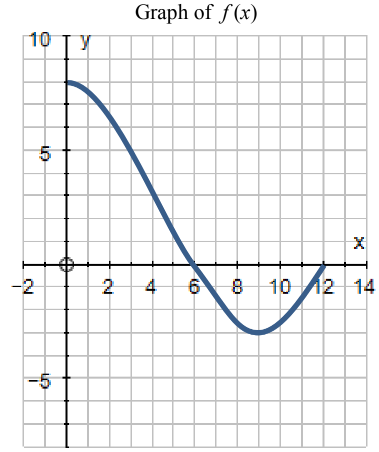
a) Use MRAM to approximate $H(10)$ using 4 equal subdivisions.

b) When is $H(x)$ decreasing? Justify your response.

c) If the average rate of change of $H(x)$ on [2, 10] is k, what is the value of $\int_2^{10} \ln(t+5)\,dt$ in terms of k.

10. [No Calculator] Let $H(x)=\int_0^x f(t)\,dt$, where f is the continuous function with domain [0, 12] shown below.

Graph of $f(x)$



a) Find $H(0)$

b) Is $H(12)$ positive or negative? Explain.

c) Find $H'(x)$ and use it to evaluate $H'(0)$.

d) When is $H(x)$ increasing? Justify your answer.

e) Find $H''(x)$.

f) When is $H(x)$ concave up? Justify your answer.

g) At what x-value does $H(x)$ achieve its maximum value? Justify your answer.

11. [No Calculator] If $\int_a^b f(x)\,dx = a + 2b$, then $\int_a^b [f(x)+3]\,dx =$

A $a + 2b + 3$

B $3b - 3a$

C $4a - b$

D $5b - 2a$

E $5b - 3a$

12. [No Calculator] Let $f(x) = \int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

A none

B 0.5

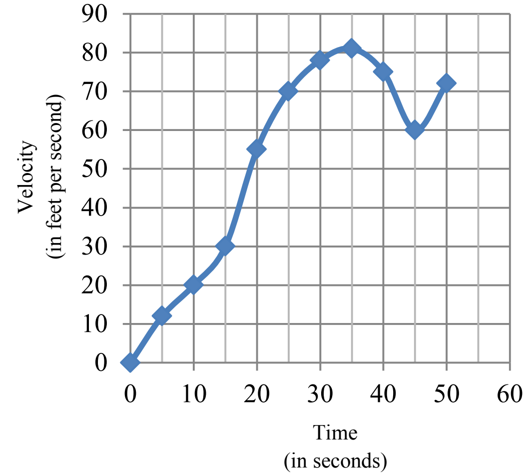
C 1.5

D 2

E 3

13. [Calculator] If $f(x) = \int_a^x \ln(2+\sin t)\,dt$, and $f(3) = 4$, what does $f(5) = ?$

Time (in seconds)	$v(t)$ (in ft/sec)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72



14. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \leq t \leq 50$.

c) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.