Name: _____ **Period:** _____

Date: _____ **AP Calc BC**

Mr. Mellina

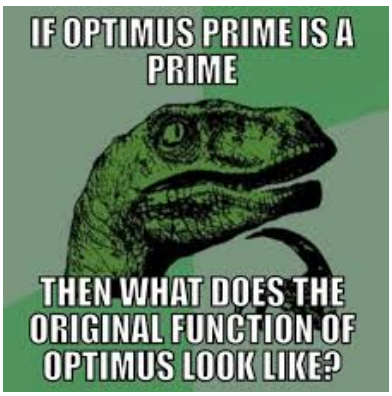
Chapter 5 Review: Logarithmic, Exponential, and other Transcendental Functions

Topics:

1. *Derivative of Natural Log Functions*
2. *Derivative of Exponential & Log Functions, Logarithmic Differentiation.*
3. *Integration with Logarithms*
4. *Integration of Exponential & Power Functions*
5. *Derivative of Inverse Functions*
6. *Derivative of Inverse Trig Functions*
7. *Integration with Inverse Trig functions.*

HW Sets

Topic 1: Chapter 5 Review Set A
Topic 2: Chapter 5 Review Set B
Topic 3: Chapter 5 Review Set C
Topic 4: Chapter 5 Review Set D
Topic 5: Chapter 5 Review Set E
Topic 6: Chapter 5 Review Set F
Topic 7: Chapter 5 Review Set G



Topic 1: Derivative of Natural Logarithmic Functions (Day 1)

These problems were selected from the Review Exercises on pages 400-402.

THEOREM 5.3 Derivative of the Natural Logarithmic Function

Let u be a differentiable function of x.

1. $\frac{d}{dx}[\ln x] = \frac{1}{x}, \quad x > 0$
2. $\frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx} = \frac{u'}{u}, \quad u > 0$

In exercises 9-16, find the derivative of the function.

9. $g(x) = \ln\sqrt{2x}$

10. $f(x) = \ln(3x^2 + 2x)$

11. $f(x) = x\sqrt{\ln x}$

12. $f(x) = [\ln(2x)]^3$

13. $y = \ln \sqrt{\frac{x^2+4}{x^2-4}}$

14. $y = \ln \frac{4x}{x-6}$

15. $y = \frac{1}{\ln(1-7x)}$

16. $y = \frac{\ln 5x}{1-x}$

In Exercises 17 & 18, find an equation of the tangent line to the graph of the function at the given point.

17. $y = \ln(2+x) + \frac{2}{2+x}$, $(-1, 2)$

18. $y = 2x^2 + \ln x^2$, $(1, 2)$

Topic 2: Derivative of Exponential & Logarithmic Functions, Logarithmic Differentiation (Day 2)

These problems were selected from the Review Exercises on pages 400-402.

THEOREM 5.11 DERIVATIVES OF THE NATURAL EXPONENTIAL FUNCTION

Let u be a differentiable function of x.

1. $\frac{d}{dx}[e^x] = e^x$
2. $\frac{d}{dx}[e^u] = e^u \frac{du}{dx}$

For exercises 47-52, find the derivative.

47. $g(t) = t^2 e^t$

48. $g(x) = \ln \frac{e^x}{1+e^x}$

49. $y = \sqrt{e^{2x} + e^{-2x}}$

50. $h(z) = e^{-z^2/2}$

51. $g(x) = \frac{x^3}{e^{2x}}$

52. $y = 3e^{-3/t}$

In Exercises 53 & 54, find an equation of the tangent line to the graph of the function at the given point.

53. $f(x) = e^{6x},\ (0, 1)$

54. $h(x) = -xe^{2-x},\ (2, -2)$

In Exercises 55 & 56, find the extrema and points of inflection (if any exist) of the function.

55. $f(x) = (x + 1)e^{-x}$

THEOREM 5.13 DERIVATIVES FOR BASES OTHER THAN *e*

Let a be a positive real number $(a \neq 1)$ and let u be a differentiable function of x.

1. $\frac{d}{dx}[a^x] = (\ln a)a^x$
2. $\frac{d}{dx}[a^u] = (\ln a)a^u \frac{du}{dx}$
3. $\frac{d}{dx}[\log_a x] = \frac{1}{(\ln a)x}$
4. $\frac{d}{dx}[\log_a u] = \frac{1}{(\ln a)u}\frac{du}{dx}$

In Exercises 75-82, find the derivative of the function.

75. $f(x) = 3^{x-1}$

76. $f(x) = 5^{3x}$

77. $g(t) = \frac{2^{3t}}{t^2}$

78. $f(x) = x(4^{-3x})$

79. $g(x) = \log_3 \sqrt{1-x}$

80. $h(x) = \log_5 \frac{x}{x-1}$

81. $y = x^{2x+1}$

82. $y = (3x+5)^x$

Topic 3: Integration with Logarithms (Day 3)

These problems were selected from the Review Exercises on pages 400-402.

THEOREM 5.5 LOG RULE FOR INTEGRATION

Let u be a differentiable function of x.

1. $\int \frac{1}{x}\,dx = \ln|x| + C$
2. $\int \frac{1}{u}\,du = \ln|u| + C$

INTEGRALS OF THE SIX BASIC TRIGONOMETRIC FUNCTIONS

$\int \sin u\,du = -\cos u + C$

$\int \cos u\,du = \sin u + C$

$\int \tan u\,du = -\ln|\cos u| + C$

$\int \cot u\,du = \ln|\sin u| + C$

$\int \sec u\,du = \ln|\sec u + \tan u| + C$

$\int \csc u\,du = -\ln|\csc u + \cot u| + C$

In exercises 21-26, find the indefinite integral.

21. $\int \frac{1}{7x-2}\,dx$

22. $\int \frac{x^2}{x^3+1}\,dx$

23. $\int \frac{\sin x}{1+\cos x}\,dx$

24. $\int \frac{\ln \sqrt{x}}{x} dx$

25. $\int \frac{x^2-6x+1}{x^2+1} dx$

26. $\int \frac{dx}{\sqrt{x}(2\sqrt{x}+5)}$

Topic 4: Integration of Exponential & Power Functions (Day 4)

These problems were selected from the Review Exercises on pages 400-402.

THEOREM 5.12 INTEGRATION RULES FOR EXPONENTIAL FUNCTIONS

Let u be a differentiable function of x.

1. $\int e^x dx = e^x + C$ 2. $\int e^u du = e^u + C$

$$\int a^x dx = \left(\frac{1}{\ln a}\right)a^x + C$$

For Exercises 57-60, 83 & 84, find the indefinite integral.

57. $\int xe^{1-x^2} dx$

58. $\int x^2 e^{x^3+1} dx$

59. $\int \frac{e^{4x}-e^{2x}+1}{e^x} dx$

60. $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}} dx$

83. $\int (x+1)5^{(x+1)^2}dx$

84. $\int \frac{2^{-1/t}}{t^2}dt$

For exercises 61 & 63, 85 & 86, evaluate the definite integral.

61. $\int_0^1 xe^{-3x^2}dx$

63. $\int_1^3 \frac{e^x}{e^x-1}dx$

85. $\int_1^2 6^x dx$

86. $\int_{-4}^0 9^{x/2}dx$

Topic 5: Derivative of Inverse Functions (Day 5)

These problems were selected from the Review Exercises on pages 400-402.

THEOREM 5.9 The Derivative of an Inverse Function

Let f be a function that is differentiable on an interval I. If f has an inverse function g, then g is differentiable at any x for which $f'(g(x)) \neq 0$. Moreover,

$$g'(x) = \frac{1}{f'(g(x))}, \quad f'(g(x)) \neq 0.$$

A proof of this theorem is given in Appendix A.

In exercises 39-42, verify that *f* has an inverse function. Then use the function *f* and the given real number *a* to find $(f^{-1})'(a)$.

39. $f(x) = x^3 + 2, \quad a = -1$

40. $f(x) = x\sqrt{x-3}, \quad a = 4$

41. $f(x) = \tan x\,, -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}, \quad a = \frac{\sqrt{3}}{3}$

42. $f(x) = \cos x\,, 0 \leq x \leq \pi, \quad a = 0$

Topic 6: Derivative of Inverse Trig Functions (Day 6)

These problems were selected from the Review Exercises on pages 400-402.

THEOREM 5.16 DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS

Let u be a differentiable function of x.

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

In exercises 99-104, find the derivative of the function.

99. $y = \csc^{-1}(2x^2)$

100. $y = \frac{1}{2}\tan^{-1}(e^{2x})$

101. $y = x\sec^{-1} x$

102. $y = \sqrt{x^2 - 4} - 2\sec^{-1}\left(\frac{x}{2}\right),\ 2 < x < 4$

103. $y = x(\sin^{-1} x)^2 - 2x + 2\sqrt{1 - x^2}\sin^{-1} x$

Topic 7: Integration with Inverse Trig (Day 7)

These problems were selected from the Review Exercises on pages 400-402.

THEOREM 5.17 INTEGRALS INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS

Let u be a differentiable function of x, and let $a > 0$.

1. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + C$

2. $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$

3. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

In exercises 105-110, find the indefinite integral.

105. $\int \frac{1}{e^{2x}+e^{-2x}}\,dx$

106. $\int \frac{1}{3+25x^2}\,dx$

107. $\int \frac{x}{\sqrt{1-x^4}}\,dx$

108. $\int \frac{1}{x\sqrt{9x^2-49}}\,dx$

109. $\int \frac{\tan^{-1}\left(\frac{x}{2}\right)}{4+x^2}\,dx$

110. $\int \frac{\sin^{-1}(2x)}{\sqrt{1-4x^2}}\,dx$