AP Calculus

5.5 Worksheet - Linearization

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Consider the function $y = \sin x$.

   a) Find the equation of the tangent line when $x = 0$.

   b) Graph both equations on your calculator in a standard viewing window.
   When would the tangent line be a good approximation for the curve? (Try zooming in at the origin)

   c) Use the tangent line to approximate sin (0.2).
   [Why is using the tangent line from part *a* a good approximation?]

2. The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is
[Why does the question ask you to use the tangent line at $x = 0$?]

   A 2.00

   B 2.03

   C 2.06

   D 2.12

   E 2.24

3. Use linearization to approximate $f(5.02)$ if $f(x) = \frac{1}{\sqrt{4+x}}$. Find the error for your approximation.

4. Suppose you were asked to determine the value of $(2.003)^4$.

a) Use linearization to approximate the value of $(2.003)^4$.

b) Use differentials to approximate the value of $(2.003)^4$

5. [With calculator] Let f be the function given by $f(x) = x^2 - 2x + 3$. The tangent line to the graph of f at $x = 2$ is used to approximate the values of $f(x)$. Which of the following is the greatest value for which the error resulting from this tangent line approximation is less than 0.5?

A 2.4

B 2.5

C 2.6

D 2.7

E 2.8

6. Find the differential dy when $dx = -0.2$ and $x = 1$, if $y = x^2 e^x$. Explain what you've found.

7. If $y = \sin(x^2 - 3)$, find dy if $x = \sqrt{3}$ and $dx = \frac{1}{10}$. Explain what you've found.

8. Without a calculator, use differentials to approximate $\sqrt[4]{19}$.

9. [Calculator Required] The range R of a projectile is $R = \frac{v_0^{\,2}}{32}(\sin 2\theta)$, where $v_0^{\,2}$ is the initial velocity in feet per second and θ is the angle of elevation. Let $v_0 = 2200$ feet per second and let θ change from 10° to 11°.
[Remember … in calculus, all degree measurements should be in radians]

a) Find the actual change in the range.

b) Use differentials to *approximate* the change in the range. What is the error in your approximation?

10. The radius of a ball bearing is measured to be 0.7 inch. If the measurement is correct to within 0.01 inch, estimate the error in the volume of the ball bearing.

AP Calculus

5.1 Worksheet - Extreme Values of Functions

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

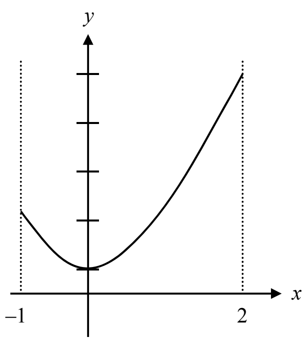
1. Earlier this year we had the Intermediate Value Theorem (IVT) and now we have the Extreme Value Theorem (EVT). The *hypothesis* of each theorem is what is needed to apply each theorem.

a) What is the hypothesis of the IVT (i.e. what is needed to apply the IVT) ?

b) What is the hypothesis of the EVT?

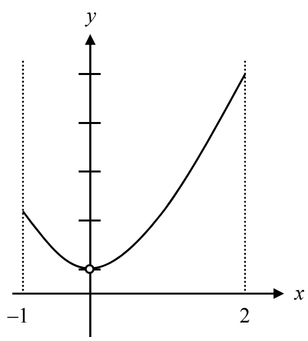
2. Using the graphs provided, find the minimum and maximum values on the given interval. *If there is no maximum or minimum value*, explain which part of the hypothesis of the Extreme Value Theorem is not satisfied.

(a) [–1, 2]



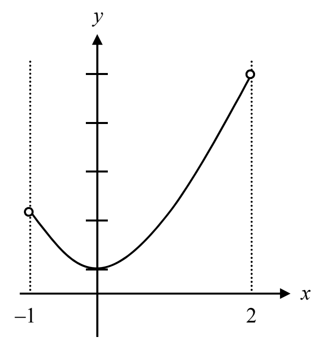
Max: _________
Min: _________

(b) [–1, 2]



Max: _________
Min: _________

(c) (–1, 2)



Max: _________
Min: _________

3. When looking for extrema, where do you find the candidates for the “candidates test”?

Each of the following statements is NOT ALWAYS TRUE. Explain/Show why each statement is false.

4. If $f'(5) = 0$, then there is a maximum or a minimum at $x = 5$.

5. If $x = 2$ is a critical number, then $f'(2) = 0$.

6. An extrema occurs at every critical number.

7. If m is a local minimum and M is a local maximum of a continuous function, then $m < M$.

8. If f is a continuous, decreasing function on [0, 10] with a critical point at (4, 2), which of the following statements MUST BE FALSE?

A $f(10)$ is an absolute minimum of f on [0, 10]
B $f(4)$ is neither a relative maximum nor a relative minimum
C $f'(4)$ does not exist.
D $f'(4)=0$
E $f'(4)<0$

9. Find the extrema on each interval and where they occur … Use a "candidates test".

a) $f(x)=\frac{1}{x}+\ln x$ when $0.5\le x\le 4$

b) $g(x)=\ln(x+1)$ when $0\le x\le 3$

c) $k(x)=x^{2/5}$ when $-3\le x<1$

10. Find the extrema of $h(\theta)=2\sin\theta-\cos(2\theta)$ for $0\le\theta\le 2\pi$. Use your graphing calculator to investigate first.

AP Calculus

5.2 Worksheet - Increasing/Decreasing Functions & Critical Points

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. State the hypothesis of each of the following theorems:

   a) IVT

   b) EVT

   c) MVT

2. State the MVT two different ways …

   a) … in words

   b) … algebraically

3. For each of the following, (a) state whether or not the function satisfies the hypotheses of the MVT on the given interval, and (b) if it does, find the value of *c* that the MVT guarantees.

   a) $f(x) = -2x^2 + 14x - 12$ on the interval [1, 6]

   b) $h(x) = x^{1/3}$ on [–1, 1]

4. When a trucker came to his second toll booth in a 169-mile stretch of road, he handed in a ticket stub that was stamped 2 hours earlier. The trucker was cited for speeding. Why?

5. Suppose $f(x)$ is a differentiable function on the interval [–7, 1] such that $f(-7) = 4$ and $f(1) = -1$.

   a) *Explain* why f must have at least one value in the interval (–7, 1), where the function equals 2.

   b) *Explain* why there must be at least one point in the interval (–7, 1) whose derivative is $-\frac{5}{8}$.

6. Make a sign chart for the following functions:

a) $f(x)=(x-3)^2(x+4)(7-x)$

b) $g(x)=\dfrac{5(2x-7)}{(x+1)(3x-5)}$

7. Summarize how we will use calculus to determine whether a function is increasing or decreasing.

8. For each function, determine where the function is increasing or decreasing. Then, find the x value of any relative extrema. *Justify ALL answers.*

a) $h(x)=\dfrac{2}{x}$

b) $f(x)=x^3-6x^2+15$

c) $k(x)=\dfrac{-x}{x^2+4}$

9. [Calculator] The Profit P in dollars made by a fast food restaurant selling x hamburgers is given by

$$P = 2.44x - \frac{x^2}{20000} - 5000, \quad 0 \le x \le 35000.$$

a) Find the intervals on which P is increasing or decreasing. [Use Calculus!]

b) Find the maximum profit.

10. If g is a differentiable function such that $g(x) < 0$ for all real numbers x, and if $f'(x) = (x^2 - 9)g(x)$, which of the following is true?

A) f has a relative maximum at $x = -3$ and a relative minimum at $x = 3$.

B) f has a relative minimum at $x = -3$ and a relative maximum at $x = 3$.

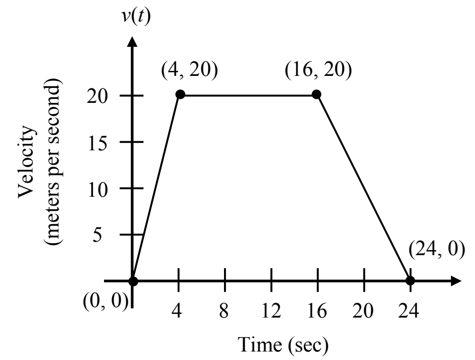
C) f has relative minima at $x = -3$ and at $x = 3$.

D) f has relative maxima at $x = -3$ and at $x = 3$.

E) It cannot be determined if f has any relative extrema.

11. A 216-m^2 pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?

12. A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph below.

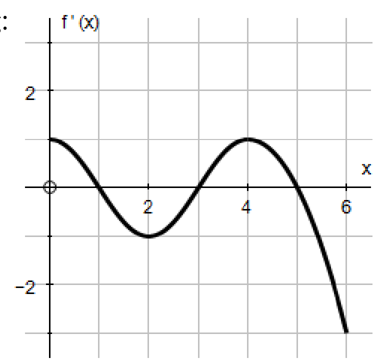


a) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

b) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

c) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

13. Use the graph of $f'(x)$ defined on [0, 6] provided below to estimate the following:



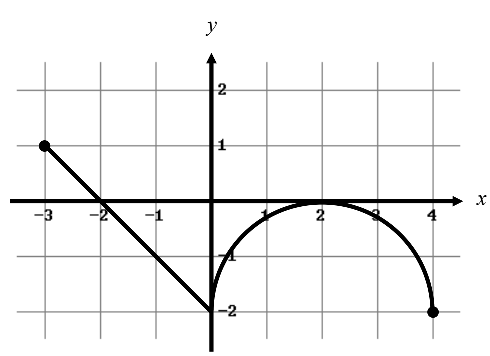
a) When is f increasing? When is f decreasing? Justify your response.

b) Determine the x-coordinates of all local extrema. Justify your response.

14. [No Calculator Allowed] Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown below.

a) On what intervals, if any, is f increasing? Decreasing? Justify your answer.

b) Find all values of x for which f assumes a relative maximum. Justify your answer.



Graph of f'

AP Calculus

5.3 Worksheet - Connecting f' and f'' with the Graph of f

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Complete each statement with the correct word.

   a) When f' is ______________________, the graph of f is increasing,.

   b) When f' is ______________________, the graph of f is decreasing,.

   c) When f'' is ______________________, the graph of f is concave upward.

   d) When f'' is ______________________, the graph of f is concave downward.

   e) When f' is ______________________, the graph of f is concave upward.

   f) When f' is ______________________, the graph of f is concave downward.

2. Use the function $y = 3x - x^3 + 5$. [No calculator allowed]

   a) Where is the function increasing? Justify your response.

   b) Where is the function decreasing? Justify your response.

   c) Where is the function concave up? Justify your response.

   d) Where is the function concave down? Justify your response.

   e) Where are the point(s) of inflection? Justify your response.

   f) Find ALL extrema and justify your response.

   g) Create a *sketch* of the function using the information you have found from *a – f*.

3. Find all local extrema of the function and justify your response using the 2nd derivative test.

$y = -2x^3 + 6x^2 - 3$

4. Determine the intervals on which the graph of each function is concave up or concave down and determine all points of inflection. Justify your responses.

a) $y = \frac{1}{20}x^5 + \frac{1}{4}x^4 - \frac{3}{2}x^3 - \frac{27}{2}x^2 + x - 4$

b) $y = 2x^{1/5} + 3$

5. If f is continuous on [0, 3] and satisfies the following:

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3
$f(x)$	0	+	2	+	0	–	–2
$f'(x)$	3	+	0	–	DNE	–	–3
$f''(x)$	0	–	–1	–	DNE	–	0

a) Find the absolute extrema of f and where they occur. Justify your response.

b) Find any points of inflection. Justify your response.

c) Sketch a possible graph of f.

6. Let f be a function that is continuous on the interval [0, 4]. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

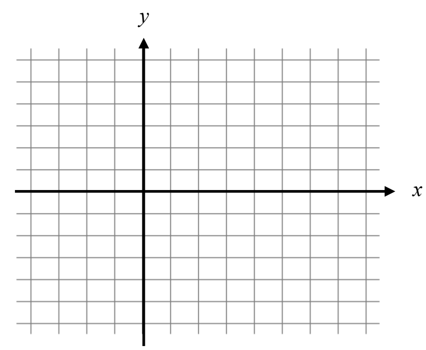
x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

a) Describe the behavior of $f(x)$ in each interval using the information above.

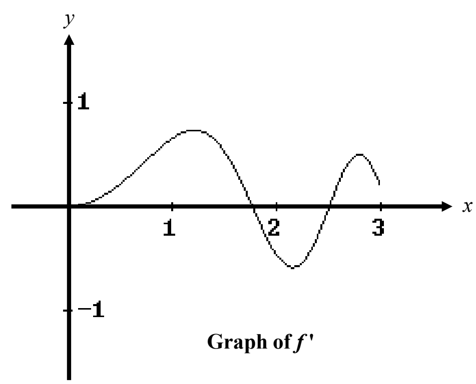
x	$0 < x < 1$	$1 < x < 2$	$2 < x < 3$	$3 < x < 4$
$f(x)$				

b) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

c) On the axes provided, sketch the graph of a function that has all the characteristics of f.



7. [Calculator Required] Let f be a function defined for $x \geq 0$ with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{(-x/4)} \sin(x^2)$. The graph of $y = f'(x)$ is shown below.

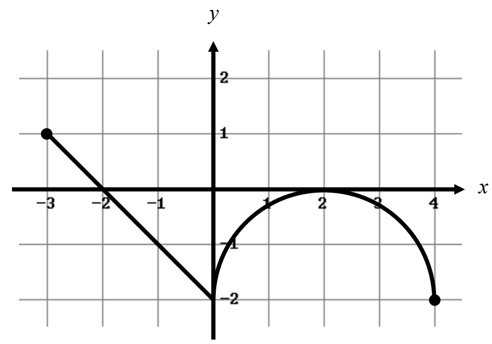


Graph of f'

a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.

b) Write an equation for the line tangent to the graph of f at the point (2, 5.623).

8. [No Calculator Allowed] Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown below.



Graph of f'

a) On what intervals, if any, is f increasing? Decreasing? Justify your answer.

b) Find all values of x for which f assumes a relative maximum. Justify your answer.

c) Where is the graph of f concave up? concave down? Justify your answers.

d) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

e) Find an equation for the line tangent to the graph of f at the point (0, 3).

f) Sketch a possible graph of f.