Name: ____________________________ **Period: ________**

Date: _______________ **AP Calc BC**

Mr. Mellina

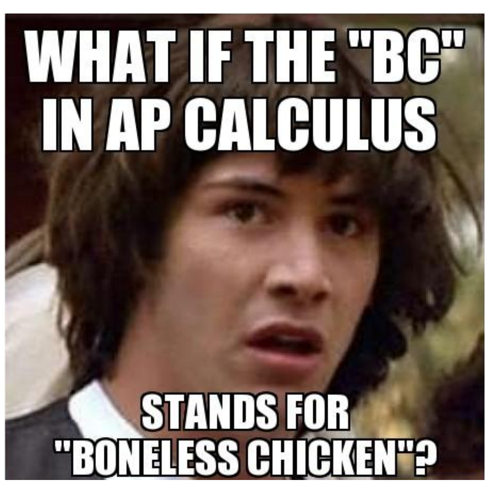
Chapter 3 Review:
Applications of Derivatives

Topics:

1. *Extreme Value Theorem*
2. *Rolle's Theorem*
3. *Mean Value Theorem*
4. *Intervals on Which a Function is Increasing or Decreasing*
5. *1st Derivative Test for Relative Extrema & Linearization*
6. *Motion Along a Line*
7. *Points of Inflection*
8. *2nd Derivative Test for Relative Extrema*
9. *Curve Sketching*
10. *Optimization*

HW Sets

Topics 1-3: Chapter 3 Review Set A
Topics 4-6: Chapter 3 Review Set B
Topics 7-9: Chapter 3 Review Set C
Topic 10: Chapter 3 Review Set D



Topic 1: Extreme Value Theorem (Day 1)

These problems were selected from the Review Exercises on pages 242-246.

> **THEOREM 3.1 The Extreme Value Theorem**
>
> If f is continuous on a closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.

In exercises 1-7, find the absolute extrema of the function on the closed interval.

1. $f(x) = x^2 + 5x, \ [-4, 0]$

3. $f(x) = \sqrt{x} - 2, \ [0, 4]$

5. $f(x) = \frac{4x}{x^2+9}, \ [-4, 4]$

7. $f(x) = 2x + 5\cos x, \ [0, 2\pi]$

Topic 2: Rolle's Theorem (Day 1)

These problems were selected from the Review Exercises on pages 242-246.

THEOREM 3.3 Rolle's Theorem

Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If $f(a) = f(b)$, then there is at least one number c in (a, b) such that $f'(c) = 0$.

In exercises 9 & 11, Determine whether Rolle's theorem can be applied to *f* on the closed interval [*a, b*]. If Rolle's Theorem can be applied, find all values of *c* in the open interval (*a, b*) such that $f'(c) = 0$. If Rolle's Theorem cannot be applied, explain why not.

9. $f(x) = x^3 - 3x - 6, \ [-1, 2]$

11. $f(x) = \dfrac{x^2}{1-x^2}, \ [-2, 2]$

Topic 3: Mean Value Theorem (Day 1)

These problems were selected from the Review Exercises on pages 242-246.

THEOREM 3.4 The Mean Value Theorem

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

In exercises 13-19, Determine whether the Mean Value Theorem can be applied to *f* on the closed interval [*a, b*]. If the Mean Value Theorem can be applied, find all values of *c* in the open interval (*a, b*) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$. If the Mean Value Theorem cannot be applied, explain why not.

13. $f(x) = x^{2/3}$, $[1, 8]$

15. $f(x) = \frac{1}{x}$, $[1, 4]$

17. $f(x) = x - \cos x$, $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

19. $f(x) = \frac{1}{x^2}$, $[-2, 1]$

Topic 4: Intervals on Which a Function is Increasing or Decreasing (Day 2)

These problems were selected from the Review Exercises on pages 242-246.

> **THEOREM 3.5 Test for Increasing and Decreasing Functions**
>
> Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).
>
> **1.** If $f'(x) > 0$ for all x in (a, b), then f is increasing on $[a, b]$.
>
> **2.** If $f'(x) < 0$ for all x in (a, b), then f is decreasing on $[a, b]$.
>
> **3.** If $f'(x) = 0$ for all x in (a, b), then f is constant on $[a, b]$.

In exercises 21-25: Find the open intervals on which the function is increasing or decreasing.

21. $f(x) = x^2 + 3x - 12$

23. $f(x) = (x-1)^2(2x-5)$

25. $h(x) = \sqrt{x}(x-3),\ x > 0$

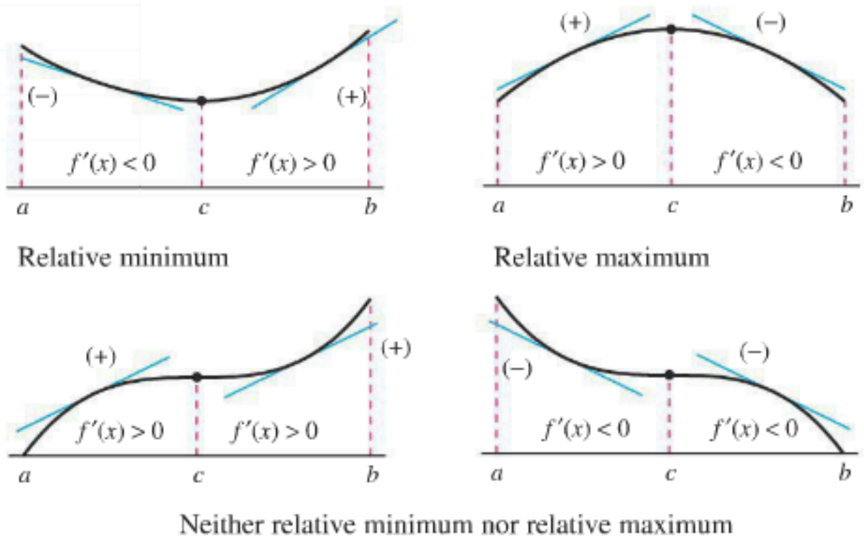
Topic 5: Applying the 1st Derivative Test & Linearization (Day 2)

These problems were selected from the Review Exercises on pages 242-246.

THEOREM 3.6 The First Derivative Test

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

1. If $f'(x)$ changes from negative to positive at c, then f has a *relative minimum* at $(c, f(c))$.
2. If $f'(x)$ changes from positive to negative at c, then f has a *relative maximum* at $(c, f(c))$.
3. If $f'(x)$ is positive on both sides of c or negative on both sides of c, then $f(c)$ is neither a relative minimum nor a relative maximum.



In exercises 27-33: Find the critical numbers of *f*, if any. Then find the open intervals on which the function is increasing or decreasing. Finally, apply the 1st Derivative Test to identify all relative extrema.

27. $f(x) = x^2 - 6x + 5$

29. $f(t) = \frac{1}{4}t^4 - 8t$

31. $f(x) = \frac{x+4}{x^2}$

33. $f(x) = \cos x - \sin x\,,\ \ (0, 2\pi)$

These problems were selected from the Exercises on page 240.

In exercises 5-9, find the tangent line approximation *L* to the graph of *f* at the given x = 2. Use *L* to approximate the given values. Then identify if this approximation is an under or over approximation.

5. $f(x) = x^2, f(1.9) \approx$

7. $f(x) = x^5, f(2.01) \approx$

9. $f(x) = \sin x, f(2.1) \approx$

Topic 6: Motion Along a Line (Day 2)

These problems were selected from the Review Exercises on pages 242-246.

In exercises 35 & 36, the function $s(t)$ describes the motion of a particle along a line. Find the velocity function of the particle for any time $t \geq 0$. Identify the time interval(s) on which the particle is moving in a positive direction. Identify the time interval(s) on which the particle is moving in a negative direction. Identify the time(s) at which the particle changes direction.

35. $s(t) = 3t - 2t^2$

36. $s(t) = 6t^3 - 8t + 3$

Topic 7: Concavity and Points of Inflection (Day 3)

These problems were selected from the Review Exercises on pages 242-246.

THEOREM 3.7 Test for Concavity

Let f be a function whose second derivative exists on an open interval I.

1. If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.
2. If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

A proof of this theorem is given in Appendix A.

THEOREM 3.8 Points of Inflection

If $(c, f(c))$ is a point of inflection of the graph of f, then either $f''(c) = 0$ or $f''(c)$ does not exist.

In exercises 37-41, find the points of inflection and discuss the concavity of the graph of the function.

37. $f(x) = x^3 - 9x^2$

39. $g(x) = x\sqrt{x+5}$

41. $f(x) = x + \cos x$, $[0, 2\pi]$

Topic 8: 2nd Derivative Test for Extrema (Day 3)

These problems were selected from the Review Exercises on pages 242-246.

THEOREM 3.9 Second Derivative Test

Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.

1. If $f''(c) > 0$, then f has a relative minimum at $(c, f(c))$.

2. If $f''(c) < 0$, then f has a relative maximum at $(c, f(c))$.

If $f''(c) = 0$, then the test fails. That is, f may have a relative maximum, a relative minimum, or neither. In such cases, you can use the First Derivative Test.

In exercises 43-47, find all relative extrema of the function. Use the Second Derivative Test where applicable.

43. $f(x) = (x + 9)^2$

45. $g(x) = 2x^2(1 - x^2)$

47. $f(x) = 2x + \frac{18}{x}$

Topic 9: Curve Sketching (Day 3)

These problems were selected from the Review Exercises on pages 242-246.

In exercises 49 & 50, sketch the graph of a function *f* having the given characteristics.

49. $f(0) = f(6) = 0$

$f'(3) = f'(5) = 0$

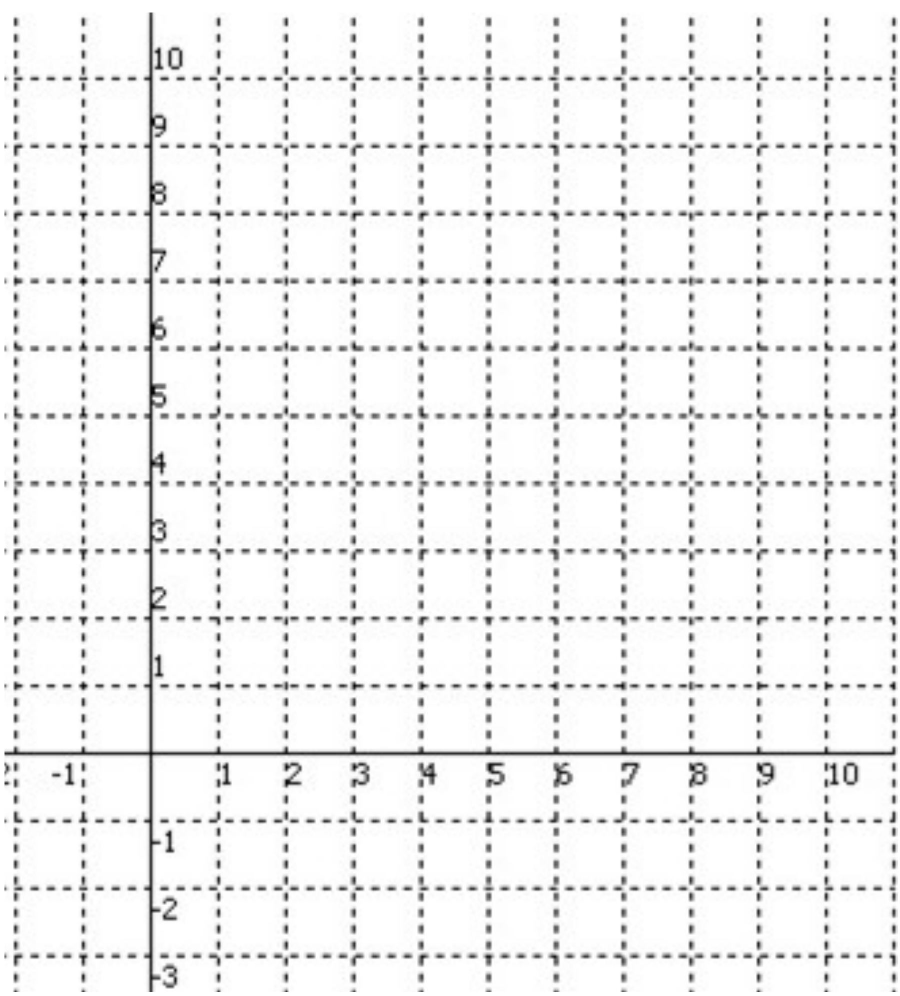
$f'(x) > 0$ for $x < 3$

$f'(x) > 0$ for $3 < x < 5$

$f'(x) < 0$ for $x > 5$

$f''(x) < 0$ for $x < 3$ or $x > 4$

$f''(x) > 0$ for $3 < x < 4$



50. $f(0) = 4$, $f(6) = 0$

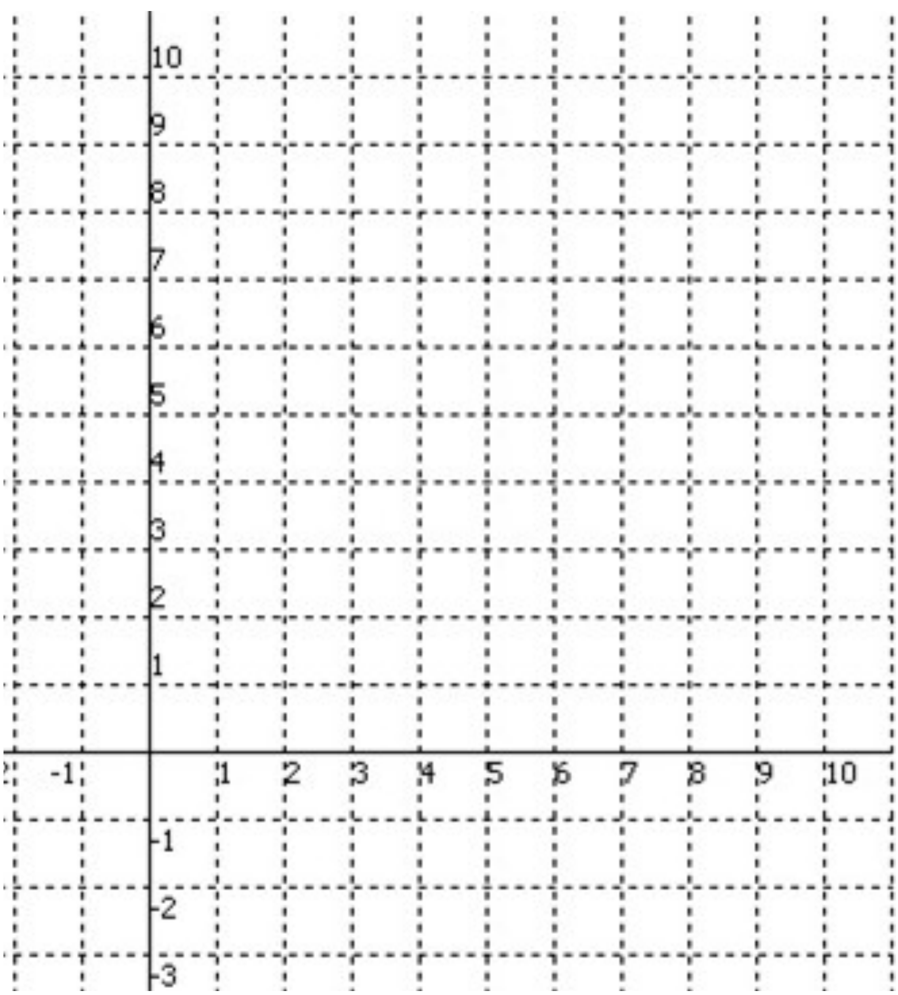
$f'(x) < 0$ for $x < 2$ or $x > 4$

$f'(2)$ does not exist.

$f'(4) = 0$

$f'(x) > 0$ for $2 < x < 4$

$f''(x) < 0$ for $x \neq 2$



Topic 10: Optimization (Day 4)

These problems were selected from the Review Exercises on pages 242-246.

79. Find two positive numbers such that the sum of twice the first number and three times the second number is 216 and the product is a maximum.

80. Find the point on the graph of $f(x) = \sqrt{x}$ that is closest to the point (6, 0).

81. A rancher has 400 feet of fencing with which to enclose two adjacent rectangular corrals (see figure). What dimensions should be used so that the enclosed area will be a maximum?