Multiple-Choice Questions on the Fundamental Theorem of Calculus

1. 1969 BC12

If $F(x) = \int_0^x e^{-t^2}\,dt$, then $F'(x) =$

(A) $2xe^{-x^2}$ (B) $-2xe^{-x^2}$ (C) $\dfrac{e^{-x^2+1}}{-x^2+1} - e$ (D) $e^{-x^2} - 1$ (E) e^{-x^2}

2. 1969 BC22

If $f(x) = \int_0^x \frac{1}{\sqrt{t^3+2}}\,dt$, which of the following is FALSE?

(A) $f(0) = 0$
(B) f is continuous at x for all $x \geq 0$
(C) $f(1) > 0$
(D) $f'(1) = \dfrac{1}{\sqrt{3}}$
(E) $f(-1) > 0$

3. 1973 AB20

If F and f are continuous functions such that $F'(x) = f(x)$ for all x, then $\int_a^b f(x)\,dx$ is

(A) $F'(a) - F'(b)$
(B) $F'(b) - F'(a)$
(C) $F(a) - F(b)$
(D) $F(b) - F(a)$
(E) none of the above

4. 1973 BC45

Suppose $g'(x) < 0$ for all $x \geq 0$ and $F(x) = \int_0^x t\,g'(t)\,dt$ for all $x \geq 0$. Which of the following statements is FALSE?

(A) F takes on negative values.
(B) F is continuous for all $x > 0$.
(C) $F(x) = x\,g(x) - \int_0^x g(t)\,dt$
(D) $F'(x)$ exists for all $x > 0$.
(E) F is an increasing function.

Special Focus: The Fundamental Theorem of Calculus

5. 1985 AB42

$$\frac{d}{dx}\int_2^x \sqrt{1+t^2}\,dt =$$

(A) $\dfrac{x}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}-5$ (C) $\sqrt{1+x^2}$ (D) $\dfrac{x}{\sqrt{1+x^2}}-\dfrac{1}{\sqrt{5}}$

(E) $\dfrac{1}{2\sqrt{1+x^2}}-\dfrac{1}{2\sqrt{5}}$

6. 1988 AB13

If the function f has a continuous derivative on $[0,c]$, then $\int_0^c f'(x)\,dx =$

(A) $f(c)-f(0)$ (B) $|f(c)-f(0)|$ (C) $f(c)$ (D) $f(x)+c$

(E) $f''(c)-f''(0)$

7. 1988 AB25

For all $x>1$, if $f(x)=\int_1^x \frac{1}{t}\,dt$, then $f'(x)=$

(A) 1 (B) $\dfrac{1}{x}$ (C) $\ln x-1$ (D) $\ln x$ (E) e^x

8. 1988 BC14

If $F(x)=\int_1^{x^2}\sqrt{1+t^3}\,dt$, then $F'(x)=$

(A) $2x\sqrt{1+x^6}$ (B) $2x\sqrt{1+x^3}$ (C) $\sqrt{1+x^6}$ (D) $\sqrt{1+x^3}$

(E) $\int_1^{x^2}\dfrac{3t^2}{2\sqrt{1+t^3}}\,dt$

9. 1993 AB41

$\dfrac{d}{dx}\int_0^x \cos(2\pi u)\,du$ is

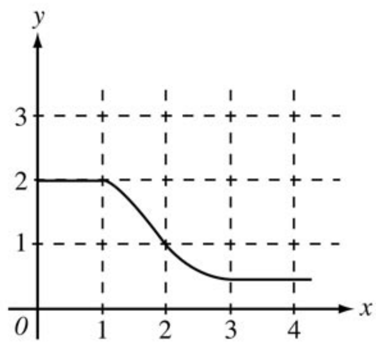
(A) 0 (B) $\dfrac{1}{2\pi}\sin x$ (C) $\dfrac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$

10. 1993 BC41

Let $f(x)=\int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

(A) For no value of x (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

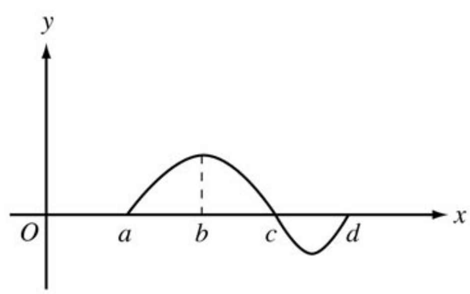
11. 1997 AB78



The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x)=f(x)$, then $F(3)-F(0)=$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

12. 1997 BC22



The graph of f is shown in the figure above. If $g(x)=\int_a^x f(t)\,dt$, for what value of x does $g(x)$ have a maximum?

(A) a (B) b (C) c (D) d

(E) It cannot be determined from the information given.

Special Focus: The Fundamental Theorem of Calculus

13. 1997 BC88

Let $f(x) = \int_0^{x^2} \sin t \, dt$. At how many points in the closed interval $\left[0, \sqrt{\pi}\right]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

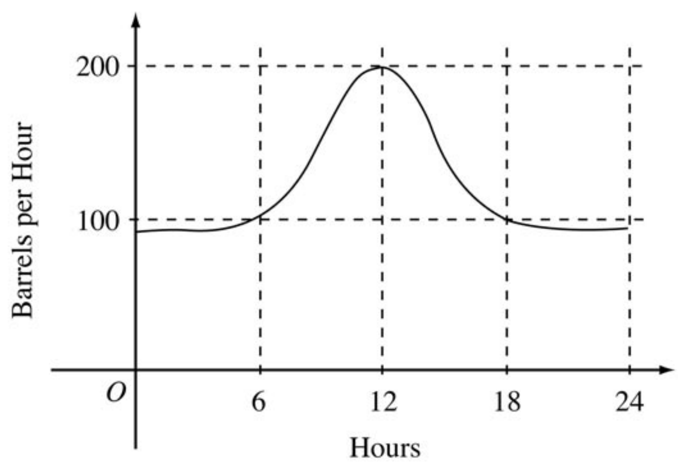
(A) Zero
(B) One
(C) Two
(D) Three
(E) Four

14. 1997 BC89

If f is the antiderivative of $\frac{x^2}{1+x^5}$ such that $f(1) = 0$, then $f(4) =$

(A) -0.012 (B) 0 (C) 0.016 (D) 0.376 (E) 0.629

15. 1998 AB9



The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

16. 1998 AB11

If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

17. 1998 AB15

If $F(x) = \int_0^x \sqrt{t^3 + 1}\,dt$, then $F'(2) =$

(A) –3 (B) –2 (C) 2 (D) 3 (E) 18

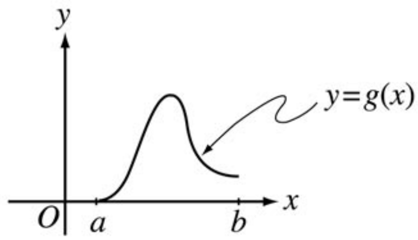
18. 1998 AB88

Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$ then $F(9) =$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

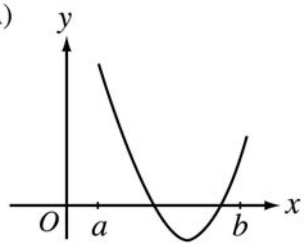
Special Focus: The Fundamental Theorem of Calculus

19. 1998 BC88

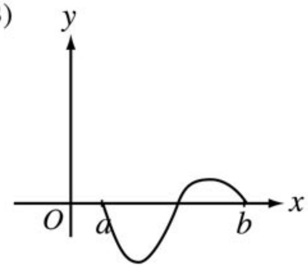


Let $g(x) = \int_a^x f(t)\,dt$, where $a \le x \le b$. The figure above shows the graph of g on $\left[a,b\right]$. Which of the following could be the graph of f on $\left[a,b\right]$?

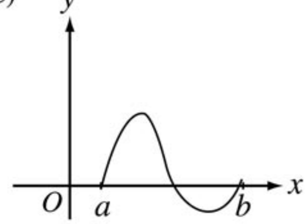
(A)



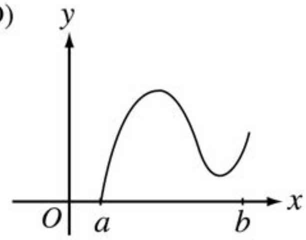
(B)



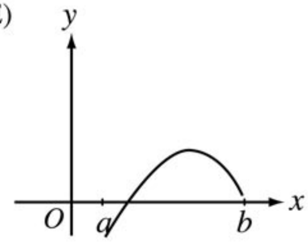
(C)



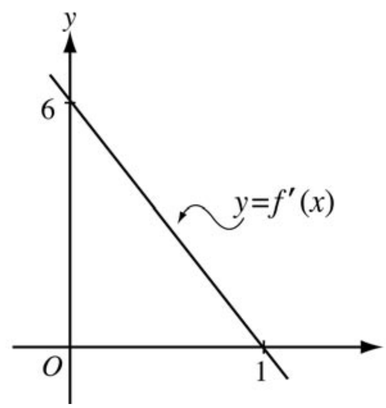
(D)



(E)



20. 2003 AB22



The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

(A) 0 (B) 3 (C) 6 (D) 8 (E) 11

21. 2003 AB82/BC82

The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)\,dt$

(B) $\int_{0}^{8} r(t)\,dt$

(C) $\int_{0}^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$

(E) $\int_{0}^{2.667} r'(t)\,dt$

Special Focus: The Fundamental Theorem of Calculus

22. 2003 AB84

A pizza, heated to a temperature of 350 degrees Fahrenheit (°F) is taken out of an oven and placed in a 75°F room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

(A) 112°F (B) 119°F (C) 147°F (D) 238°F (E) 335°F

23. 2003 AB91

A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln\left(1+2^t\right)$. If the velocity of the particle is 2 at time $t = 1$ then the velocity of the particle at time $t = 2$ is

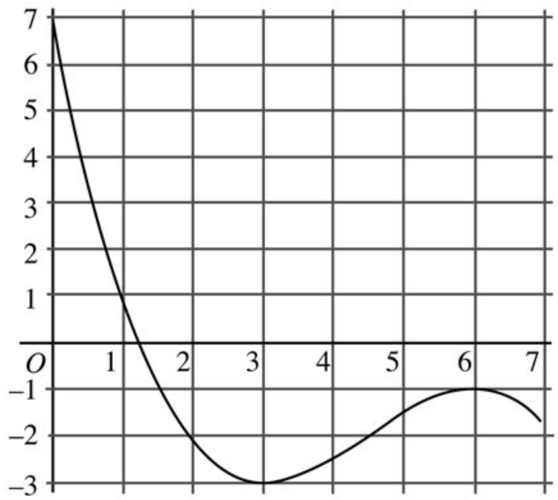
(A) 0.462 (B) 1.609 (C) 2.555 (D) 2.886 (E) 3.346

24. 2003 AB92

Let g be the function given by $g(x) = \int_0^x \sin\left(t^2\right)dt$ for $-1 \le x \le 3$. On which of the following intervals is g decreasing?

(A) $-1 \le x \le 0$
(B) $0 \le x \le 1.772$
(C) $1.253 \le x \le 2.171$
(D) $1.772 \le x \le 2.507$
(E) $2.802 \le x \le 3$

25. 2003 BC18



Graph of f

The graph of the function f shown in the figure above has horizontal tangents at $x = 3$ and $x = 6$. If $g(x) = \int_0^{2x} f(t)\,dt$, what is the value of $g'(3)$?

(A) 0 (B) –1 (C) –2 (D) –3 (E) –6

26. 2003 BC27

$$\frac{d}{dx}\left(\int_0^{x^3} \ln\left(t^2+1\right)dt\right) =$$

(A) $\frac{2x^3}{x^6+1}$ (B) $\frac{3x^2}{x^6+1}$ (C) $\ln\left(x^6+1\right)$ (D) $2x^3\ln\left(x^6+1\right)$

(E) $3x^2\ln\left(x^6+1\right)$

27. 2003 BC80

Insects destroyed a crop at the rate of $\frac{100e^{-0.1t}}{2-e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons did the insects destroy during the time interval $7 \le t \le 14$?

(A) 125 (B) 100 (C) 88 (D) 50 (E) 12

28. 2003 BC87

A particle moves along the x-axis so that at any time $t \ge 0$, its velocity is given by $v(t) = \cos\left(2 - t^2\right)$. The position of the particle is 3 at time $t = 0$. What is the position of the particle when its velocity is first equal to 0?

(A) 0.411 (B) 1.310 (C) 2.816 (D) 3.091 (E) 3.411

Multiple-Choice Question Solutions

1. (1969 BC12)

(E) By the Fundamental Theorem of Calculus, if $F(x) = \int_0^x e^{-t^2}\,dt$, then $F'(x) = e^{-x^2}$.

2. (1969 BC22)

(E) Since $f(x) = \int_0^x \frac{1}{\sqrt{t^3+2}}\,dt$, $f(-1) = \int_0^{-1} \frac{1}{\sqrt{t^3+2}}\,dt = -\int_{-1}^{0} \frac{1}{\sqrt{t^3+2}}\,dt < 0$ because the integrand is positive on the interval [−1,0]. Since $f(-1) < 0$, E is false. A is true because $f(0) = \int_0^0 \frac{1}{\sqrt{t^3+2}}\,dt = 0$. B is true because f is differentiable for all $x \ge 0$. C is true because the integrand is positive on the interval [0,1]. D is true because $f'(x) = \frac{1}{\sqrt{x^3+2}}$ by the FTC.

3. (1973 AB20)

(D) By the Fundamental Theorem of Calculus, $\int_a^b f(x)\,dx = F(b) - F(a)$ since $F'(x) = f(x)$.

4. (1973 BC45)

(E) $F'(x) = xg'(x)$ with $x \geq 0$, and $g'(x) < 0 \Rightarrow F'(x) < 0 \Rightarrow F$ is not increasing. Hence E is false. A is true because $F(1) = \int_0^1 tg'(t)\,dt < 0$ since the integrand is negative on the interval (0, 1). B is true because by the FTC, the function F is differentiable for all $x > 0$. C is true using integration by parts. D is true because $F'(x) = xg'(x)$ for all $x > 0$.

5. (1985 AB42)

(C) This is a direct application of the Fundamental Theorem of Calculus:

$$f'(x) = \sqrt{1+x^2}.$$

6. (1988 AB13)

(A) By the Fundamental Theorem of Calculus, $\int_0^c f'(x)\,dx = f(c) - f(0)$.

7. (1988 AB25)

(B) Use the Fundamental Theorem of Calculus: $f'(x) = \dfrac{1}{x}$.

8. (1988 BC14)

(A) Use the Fundamental Theorem of Calculus and the chain rule:

$$F'(x) = \sqrt{1+\left(x^2\right)^3} \cdot \frac{d\left(x^2\right)}{dx} = 2x\sqrt{1+x^6}.$$

9. (1993 AB41)

(D) This is a direct application of the Fundamental Theorem.

10. (1993 BC41)

(C) Use the Fundamental Theorem and the chain rule: $f'(x) = (2x-3)e^{\left(x^2-3x\right)^2}$. Therefore $f' < 0$ for $x < \dfrac{3}{2}$ and $f' > 0$ for $x > \dfrac{3}{2}$, so f has its absolute minimum at $x = \dfrac{3}{2}$.

11. (1997 AB78)

(D) $F(3) - F(0) = \int_0^3 F'(x)dx = \int_0^3 f(x)dx = \int_0^1 f(x)dx + \int_1^3 f(x)dx = 2 + 2.3 = 4.3.$

12. (1997 BC22)

(C) By the Fundamental Theorem, $g'(x) = f(x)$. The only critical value of g on (a, d) is at $x = c$, where g' changes from positive to negative. Thus the absolute maximum for g occurs at $x = c$.

13. (1997 BC88)

(C) By the FTC and the chain rule, $f'(x) = 2x\sin(x^2)$. For the average rate of change of f we need to determine $f(0)$ and $f(\sqrt{\pi})$. We have $f(0) = 0$ and $f(\sqrt{\pi}) = \int_0^{\pi} \sin t\, dt = 2$. The average rate of change of f on the interval is therefore $\frac{2}{\sqrt{\pi}}$. See how many points of intersection there are for the graphs of $y = 2x\sin(x^2)$ and $y = \frac{2}{\sqrt{\pi}}$ on the interval $\left[0, \sqrt{\pi}\right]$. There are two.

14. (1997 BC89)

(D) Both statements below follow from the Fundamental Theorem of Calculus:

$$f(x) = \int_1^x \frac{t^2}{1+t^5}\,dt;\ f(4) = \int_1^4 \frac{t^2}{1+t^5}\,dt = 0.376, \text{ or}$$

$$f(4) = f(1) + \int_1^4 \frac{x^2}{1+x^5}\,dx = 0.376.$$

15. (1998 AB9)

(D) Let $r(t)$ be the rate of oil flow as given by the graph, where t is measured in hours. The total number of barrels is given by $\int_0^{24} r(t)dt$. This can be approximated by counting the squares below the curve and above the horizontal axis. There are approximately five squares with area 600 barrels. Thus the total is about 3,000 barrels.

16. (1998 AB11)

(A) $\int_a^b f''(x)\,dx = f'(b) - f'(a) = 0$ since the derivative is constant for a linear function. (Alternatively, $f''(x) = 0$ for a linear function, so the value of the definite integral is 0.)

17. (1998 AB15)

(D) By the Fundamental Theorem of Calculus, $F'(x) = \sqrt{x^3 + 1}$, hence $F'(2) = \sqrt{2^3 + 1} = 3$.

18. (1998 AB88)

(C) $F(9) = F(1) + \int_1^9 F'(t)\,dt = F(1) + \int_1^9 \frac{(\ln t)^3}{t}\,dt = 5.827$.

19. (1998 BC88)

(C) From the given information, $g'(x) = f(x)$. We want a graph for *f* that represents the slope of the graph of *g* in the figure. The slope of *g* is 0 at *a* and *b*. Also the slope goes from positive to negative. This is true only for the graph of *f* in figure (C).

20. (2003 AB22)

(D) By the Fundamental Theorem of Calculus,

$$f(1) = f(0) + \int_0^1 f'(x)dx = 5 + \frac{1}{2} \cdot 1 \cdot 6 = 8.$$

21. (2003 AB82/BC82)

(A) Graph $r(t)$ on the interval $0 \le t \le 8$ and solve for where $r(t) = 0$. This occurs where $t = 1.572$ and $t = 3.514$, and $r(t) < 0$ between these two values. So the altitude is decreasing for $1.572 \le t \le 3.514$. By the Fundamental Theorem of Calculus, the change in altitude over this interval is the definite integral of the rate of change of the altitude over this interval, so the change in altitude is given by the definite integral in (A).

22. (2003 AB84)

(A) Let $T(t)$ be the temperature at time *t*. Then $T'(t) = -110e^{-0.4t}$. By the Fundamental Theorem of Calculus,

$$T(5) = T(0) + \int_0^5 T'(t)\,dt = 350 + \int_0^5 -110e^{-0.4t}dt = 112.217,$$

or 112 to the nearest degree.

Special Focus: The Fundamental Theorem of Calculus

23. (2003 AB91)

(E) By the Fundamental Theorem of Calculus,

$v(2) = v(1) + \int_1^2 v'(t)\,dt = 2 + \int_1^2 \ln(1+2^t)\,dt = 3.346$ using numerical integration on the calculator.

24. (2003 AB92)

(D) By the Fundamental Theorem of Calculus, $g'(x) = \sin(x^2)$. Graph this function on the interval $-1 \le x \le 3$ and look for where the derivative is negative. This happens a bit before $x = 2$ and at approximately $x = 2.5$. Thus the answer is (D). One can check that the endpoints of the interval in (D) are indeed where $g'(x) = 0$.

25. (2003 BC18)

(C) By the Fundamental Theorem of Calculus and the chain rule, $g'(x) = f(2x)\cdot 2$. Therefore $g'(3) = 2f(6) = 2(-1) = -2$.

26. (2003 BC27)

(E) By the Fundamental Theorem of Calculus and the chain rule,

$$\frac{d}{dx}\left(\int_0^{x^3} \ln(t^2+1)\,dt\right) = \ln\left((x^3)^2+1\right)\cdot\frac{d}{dx}(x^3) = 3x^2\ln(x^6+1).$$

27. (2003 BC80)

(A) By the Fundamental Theorem of Calculus, the total change in the amount of crops is the definite integral of the rate of change in the amount of crops.

Hence the amount destroyed is $\int_7^{14} \frac{100e^{-0.1t}}{2-e^{-3t}}\,dt = 124.994$, or 125 to the nearest ton.

28. (2003 BC87)

(C) The velocity is first equal to 0 when $2 - t^2 = \frac{\pi}{2}$ or $t = \sqrt{2 - \frac{\pi}{2}} = 0.655136$.

By the Fundamental Theorem of Calculus,

$$s(0.655136) = s(0) + \int_0^{0.655136} v(t)\,dt = 3 + \int_0^{0.655136} \cos(2-t^2)\,dt = 2.816.$$