AP Calculus
4.1 Worksheet (Chain Rule)

*All work must be shown in this course for full credit. Unsupported answers may receive NO credit.*

**For questions 1 - 6, find $\frac{dy}{dx}$.**

1. $y=\cos\left(\frac{\pi}{2}-3x\right)$

2. $y=\left(x-\sqrt{x}\right)^{-2}$

3. $y=\sin^{-5}x-\cos^{3}x$

4. $y=\dfrac{3}{\sqrt{2x+1}}$

5. $y=\sin^{2}\left(3x-2\right)$

6. $y=\sqrt{\tan\left(5x\right)}$

**For questions 7 and 8, find the indicated derivative.**

7. Find $\dfrac{dr}{d\theta}$ if $r=\sqrt{\theta\sin\theta}$

8. Find $\dfrac{ds}{d\theta}$, if $s=2\theta\sqrt{\sec\theta}$

9. Find $\dfrac{d^2y}{dx^2}$ if $y=\tan(3x-1)$

10. What is the largest possible value for the slope of the curve $g(x) = \sin\left(\frac{x}{3}\right)$?

11. Find the equation of the tangent line when $x = 4$ on the function $f(x) = \sqrt{25 - x^2}$ .

12. The position of a particle moving along the $x$-axis is given by the equation $x(t) = \sqrt{1 + 4t}$ .

    a) Find $v(t)$.

    b) Find $a(t)$

    c) When is the particle stopped?

    d) Is the particle speeding up or slowing down at $t = 6$? Justify your response.

13. Suppose that $x$ is a function of $t$. Find $\frac{dy}{dt}$, if $y = \tan x$ .

14. Suppose $f$ and $g$ are differentiable functions with the values given in the table below.

| $x$ | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 2 | 5 | 5 | $e$ | $\sqrt{2}$ |
| 5 | 2 | 8 | $\pi$ | 7 |

a) If $h(x)=f(g(x))$, write an expression for $h'(x)$ and use it to find $h'(2)$.

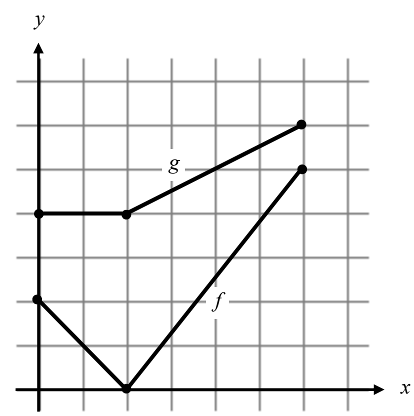
b) If $h(x)=g(f(x))$, write an expression for $h'(x)$ and use it to find find $h'(2)$.

c) If $h(x)=f(f(x))$, write an expression for $h'(x)$ and use it to find find $h'(2)$.

15. Let $r(x)=f(g(x))$ and $s(x)=g(f(x))$ where $f$ and $g$ are shown in the figure below.

a) Find $r'(1)$.

b) Find $s'(4)$.




(Challenge Question)

16. Four functions ($f$, $g$, $h$, and $j$) are continuous and differentiable for all real numbers , and some of their values (and the values of their derivatives) are given by the table below. If you know that $h(x)=f(x)\cdot g(x)$ and $j(x)=g(f(x))$, fill in the correct numbers for each blank value in the table.

| $x$ | 0 | 1 |
|---|---|---|
| $f(x)$ | 1 | |
| $g(x)$ | | 3 |
| $h(x)$ | | 0 |
| $j(x)$ | | –1 |
| $f'(x)$ | 4 | |
| $g'(x)$ | 1 | |
| $h'(x)$ | | 6 |
| $j'(x)$ | 8 | |

AP Calculus
4.4 Worksheet (Derivatives of Exponential and Logarithmic Functions)

*All work must be shown in this course for full credit. Unsupported answers may receive NO credit.*

1. Find the derivative of each of the following functions:

a) $f(x) = 7^{3x+1}$

b) $g(x) = 9^{x^2}$

c) $h(x) = e^{2x^3-5}$

d) $k(x) = \log_2(x^2 - 9)$

e) $j(x) = \log(3x+7)$

f) $m(x) = \ln(3x^5 + 8)$

2. Find $\frac{dy}{dx}$ for the following functions:

a) $y = e^{-5x}$

b) $y = xe^2 - e^{-x}$

c) $y = x^2e^x - xe^x$

d) $y = 3^{\csc x}$

e) $y = (\ln x)^2$

f) $y = \log_5 \sqrt{x}$

g) $y = \log_3(1 + x\ln x)$

h) $y = \ln 2 \cdot \log_2 x$

i)

j) $y = x^e e^x$

k) $y = \ln(\ln x)$

l) $y = \log_2\left(\frac{1}{x}\right)$

~~3. Suppose $10 = e^{xy} + x^2 + y^2$, find $\frac{dy}{dx}$.~~

4. [Calculator Required] At what point on the graph of $y = 3^x + 1$ is the tangent line parallel to $5x - y = 1$?

5. [Calculator Required] At what point on $y = 2e^x - 1$ is the tangent line perpendicular to the line $3x + y = 2$?

6. If $y = \tan u$, $u = v - \frac{1}{v}$, and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?

A $0$
B $1/e$
C $1$
D $2/e$
E $\sec^2(e)$

~~7. Use logarithmic differentiation to find the derivative of the following functions:~~

~~a) $y = x^{\ln x}$~~

b) $\cancel{y = \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}}$

~~8. Use logarithmic differentiation. If $f(x)=\left(x^2+1\right)^{(2-3x)}$, then $f'(1)$~~

9. Does $e^x$ ever equal zero? Explain.

10. If $y=x^2e^x$, find where $\frac{dy}{dx}=0$.

11. If $y=\frac{e^x}{x^3}$, find where $\frac{dy}{dx}$ is equal to zero <u>and</u> undefined.

AP Calculus
4.2 Worksheet(Implicit Differentiation)

*All work must be shown in this course for full credit. Unsupported answers may receive NO credit.*

1. Use implicit differentiation to find $\frac{dy}{dx}$.

   a) $x^2y + xy^2 = 6$

   b) $x + \tan(xy) = 0$

2. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

   a) $y^2 = x^2 + 2x$

   [redacted]

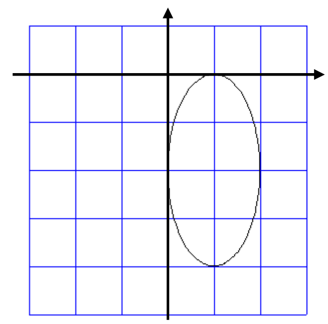
3. Find the equations of the tangent line and the normal line to the curves.

   a) $y = \sin(\pi x - y)$ at the point (1, 0)

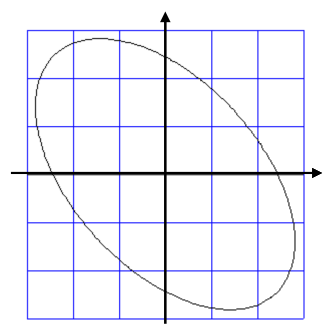
   b) $y^2(2 - x) = x^3$ at the point (1,1)

4. Find the equations of the tangent line and the normal line to the curve $x^2 + xy - y^2 = 1$ when $x = 2$.

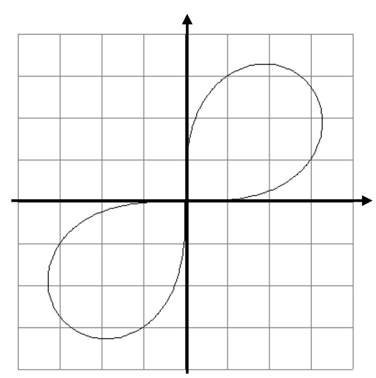
5. Find the points at which the graph of $4x^2 + y^2 - 8x + 4y + 4 = 0$ has a vertical tangent line. (Pretend the picture isn't there until *after* you have found the points!)



6. Find the point(s) (if any) of horizontal tangent lines: $x^2 + xy + y^2 = 6$
(Does your answer make sense given the picture to the right?)



7. Determine the slope of the graph of $3\left(x^2 + y^2\right)^2 = 100xy$ at the point (3, 1).
(Does your answer make sense given the picture to the right?)



8. Consider the curve defined by $2y^3 + 6x^2 y - 12x^2 + 6y = 1$.

a) Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$

b) Write an equation of each horizontal tangent line to the curve.

c) The line through the origin with slope –1 is tangent to the curve at point *P*. Find the *x* – and *y* – coordinates of point *P*.

9. The line that is normal to the curve $x^2 + 2xy - 3y^2 = 0$ at the point (1,1) intersects the curve at what other point?

AP Calculus
5.6 Worksheet(Related Rates)

*All work must be shown in this course for full credit. Unsupported answers may receive NO credit.*

OK … I couldn't find a decent looney tunes picture for the next problem, so I thought I'd just throw in this cartoon (which by the way has nothing to do with related rates!) since I found it looking for any other good pictures. Besides, poor Wile E. Coyote has been working so much this year, it's about time he finally got a good meal. ☺


1. The radius $r$ and area $A$ of a circle are related by the equation: $A = \pi r^2$
Write an equation that relates $\frac{dA}{dt}$ and $\frac{dr}{dt}$.

2. A spherical container is deflated such that its volume is decreasing at a constant rate of $3141 \text{cm}^3/\text{min}$.
[The Surface area of a sphere is $S = 4\pi r^2$ The volume of a sphere is $V = \frac{4}{3}\pi r^3$ ]

a) At what rate is the radius changing when the radius is 5 cm? Indicate units of measure.

b) At that same moment, how fast is the Surface Area changing? Indicate units of measure.

3. A pebble is dropped into a still pool and sends out a circular ripple whose radius increases at a constant rate of 4 ft/s. How fast is the area of the region enclosed by the ripple increasing at the end of 8 s? Indicate units of measure.

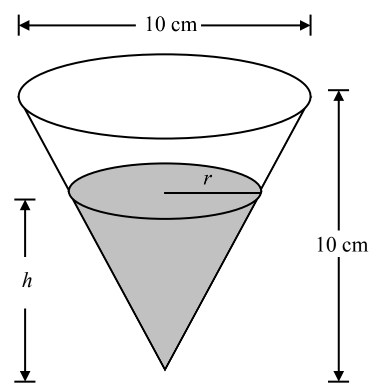
4. A 14 ft ladder is leaning against a wall. The top of the ladder is slipping down the wall at a rate of 2 ft/s.

a) How fast will the end of the ladder be moving away from the wall when the top is 6 ft above the ground? Indicate units of measure.

b) At the same moment, how fast is the angle between the ground and the ladder changing?

5. A container has the shape of an open right circular cone, as shown in the figure to the right. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth $h$ is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

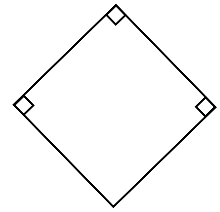
(The volume of a cone of height $h$ and radius $r$ is given by $V = \frac{1}{3}\pi r^2 h$.)




a) Find the volume $V$ of water in the container when $h$ = 5 cm. Indicate units of measure.

b) Find the rate of change of the volume of water in the container, with respect to time, when $h$ = 5 cm. Indicate units of measure.

6. A baseball diamond has the shape of a square with sides 90 feet long. Tweety is just flying around the bases, running from 2nd base (top of the diamond) to third base (left side of diamond) at a speed of 28 feet per second. When Tweety is 30 feet from third base, at what rate is Tweety's distance from home plate (bottom of diamond) changing? Indicate units of measure.



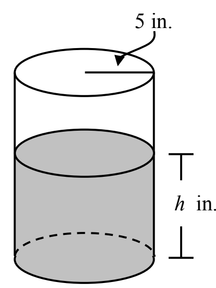
7. The radius $r$, height $h$, and volume $V$ of a right circular cylinder are related by the equation $V = \pi r^2 h$.

a) How is $\frac{dV}{dt}$ related to $\frac{dh}{dt}$ <u>if $r$ is constant</u>?

b) How is $\frac{dV}{dt}$ related to $\frac{dr}{dt}$ <u>if $h$ is constant</u>?

c) How is $\frac{dV}{dt}$ related to $\frac{dr}{dt}$ and $\frac{dh}{dt}$ if neither $r$ nor $h$ is constant?

8. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure to the right. Let $h$ be the depth of the coffee in the pot, measured in inches, where $h$ is a function of time $t$, measured in seconds. The volume $V$ of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume $V$ of a cylinder with radius $r$ and height $h$ is $V = \pi r^2 h$.)
Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

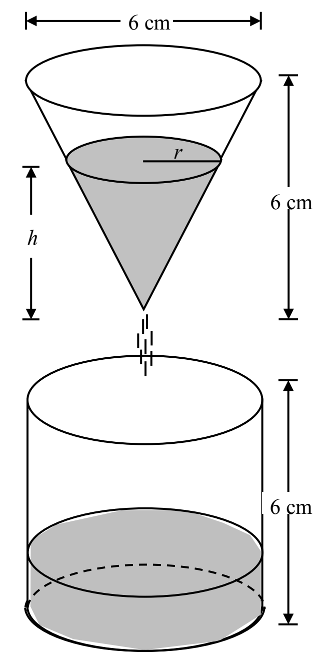



9. Sand pours out of a chute into a conical pile whose height is always one half its diameter. If the height increases at a constant rate of 4 ft/min, at what rate is sand pouring from the chute when the pile is 15 ft high? Indicate units of measure.

10. A camera man is standing 1000 feet from the launch of a rocket. As the rocket launches, the camera man must change the angle of elevation of his camera to keep the rocket in the camera's view. How fast is the angle of elevation changing when the rocket is 1 mile (5280 feet) in the air if the rocket is moving at 2300 feet per second?

11. Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of 10 in$^3$/min.

a) How fast is the level in the pot rising when the coffee in the cone is 5 in. deep?




b) How fast is the level in the cone falling at that moment?