My Big Chapter Two Review **2009ch2review.doc**

1. $\lim_{x\to 2} \frac{x^3-8}{x-2}$ $= \frac{d}{dx} x^3 \Big|_{x=2}$

$= 3x^2 \Big|_{x=2}$

$= 12$

2. $\lim_{h\to 0} \frac{\tan\left(\frac{\pi}{4}+h\right) - \tan\left(\frac{\pi}{4}\right)}{h}$ $= \frac{d}{dx} \tan x \Big|_{x=\frac{\pi}{4}}$

$= \sec^2 x \Big|_{x=\frac{\pi}{4}}$

$= 2$

3. Write the equation of the line tangent to the graph of $y = 5x - \sin x$ at $x = 2\pi$

$y(2\pi) = 10\pi$ Point $(2\pi, 18\pi)$

$y'(x) = 5 - \cos x$

$y'(2\pi) = 5 - 1 = 4$

EQUATION OF TANGENT LINE $y - 18\pi = 4(x - 2\pi)$

4. Let $f(x) = \begin{cases} 9x-4, & x \le 1 \\ 4x^2+1, & x > 1 \end{cases}$ $f'(x) = \begin{cases} 9, & x < 1 \\ 8x, & x > 1 \end{cases}$

Is this function continuous and/or differentiable at $x = 1$

TO BE CONTINUOUS

$f(1) = 5$ $\lim_{x\to 1^-} f(x) = 5$ $\lim_{x\to 1^+} f(x) = 5$

$\lim_{x\to 1} f(x) = 5 = f(1)$ HENCE, CONTINUOUS $x = 1$

$\lim_{x\to 1^-} f'(x) = 9$ $\lim_{x\to 1^+} f'(x) = 8$

HENCE, NOT DIFFERENTIABLE AT $x = 1$

5. If $\lim_{h \to 0} \frac{f(10+h)-f(10)}{h} = 25$, then which of the following may we assume to be true:

$f'(10) = 25$

I $f(10) = 25$

II $f'(10) = 25$

III f is both continuous and differentiable at $x = 10$

6. Given the following information about differentiable functions $f(x)$ and $g(x)$ at $x = 2$ determine the following:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	8	2	2π	e

(a) $\frac{d}{dx}[f(g(x))]$ at $x = 2$

$= f'(g(x))\,g'(x)\Big|_{x=2}$

$= f'(g(2))\,g'(2)$

$= f'(2)\,g'(2) = 2\pi e$

(b) $\frac{d}{dx}\left(\frac{1}{f(x)}\right)$ at $x = 2$

$= \frac{0 - f'(x)}{[f(x)]^2}\Big|_{x=2}$

$= \frac{-f'(2)}{[f(2)]^2} = \frac{-2\pi}{64}$

(c) $\frac{d}{dx}\left[\frac{g(x)}{f(x)}\right]$ at $x = 2$

$= \frac{f(x)g'(x) - g(x)f'(x)}{[f(x)]^2}\Big|_{x=2}$

$= \frac{8e - (2)(2\pi)}{64}$

7. Find $\frac{d}{dx}\left[\sin\sqrt[3]{x}\right] = \frac{d}{dx}\sin u$

$u = \sqrt[3]{x}$

$\frac{du}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$

$= \cos u \frac{du}{dx}$

$= \left[\cos\sqrt[3]{x}\right]\left[\frac{1}{3}x^{-\frac{2}{3}}\right]$

8. Find $f'(x)$ if $f(x) = \frac{2x+3}{3x+2}$

$$f'(x) = \frac{(3x+2)(2) - (2x+3)(3)}{(3x+2)^2}$$

$$= \frac{-5}{(3x+2)^2}$$

9. Given $25x^2 + 8x - 16y^2 - 4y - 9 = 0$

(a) Find $\frac{dy}{dx}$

$$\frac{d}{dx}25x^2 + \frac{d}{dx}8x - \frac{d}{dx}16y^2 - \frac{d}{dx}4y - \frac{d}{dx}9 = \frac{d}{dx}0$$

$$50x + 8 - 32y\frac{dy}{dx} - 4\frac{dy}{dx} = 0$$

$$50x + 8 = \frac{dy}{dx}(32y+4)$$

$$\frac{50x+8}{32y+4} = \frac{dy}{dx}$$

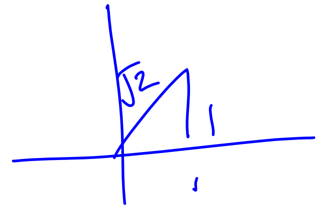
(b) Find any value(s) of x where the curve has a horizontal tangent

Horizontal tangent if $\frac{dy}{dx} = 0$, so let $50x + 8 = 0$. Hence, at $x = \frac{-8}{50}$

(c) Find the value(s) of y where the curve has a vertical tangent

Vertical tangent if $\frac{dy}{dx}$ is undefined, so let $32y + 4 = 0$. Hence, at $y = \frac{-4}{32}$

10. What is the slope of the tangent line to the graph of $y = \tan(2x)$ at $x = \frac{\pi}{8}$?



$y = \tan u$ $\quad u = 2x$

$\frac{du}{dx} = 2$

$y' = \sec^2 u \frac{du}{dx}$

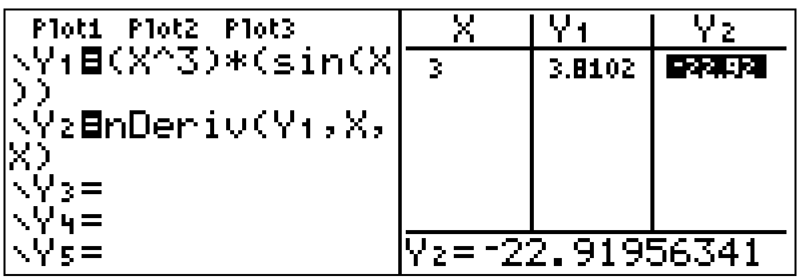
$y' = 2\sec^2(2x)$

$y'\left(\frac{\pi}{8}\right) = 2\sec^2\left(\frac{\pi}{4}\right) = 4$

Some calculator-friendly problems:

11. A particle moves along the x-axis so that any time $t \geq 0$, its velocity is given by $v(t) = t^3 \sin t$. Find the acceleration at $t = 3$ AND determine if the speed is increasing or decreasing at $t = 3$

$a(t) = v'(t)$ so $a(3) = v'(3)$



$a(3) \approx -22.91956$

$v(3) \approx 3.8102$

at $t = 3$ the speed is decreasing because $v(3) > 0$ and $a(3) < 0$

12. A pebble is thrown into a pond forming ripples whose radius increases at a rate of 4 inches/second. How fast is the area of the ripple changing when the radius is 12 inches?

$a = \pi r^2$

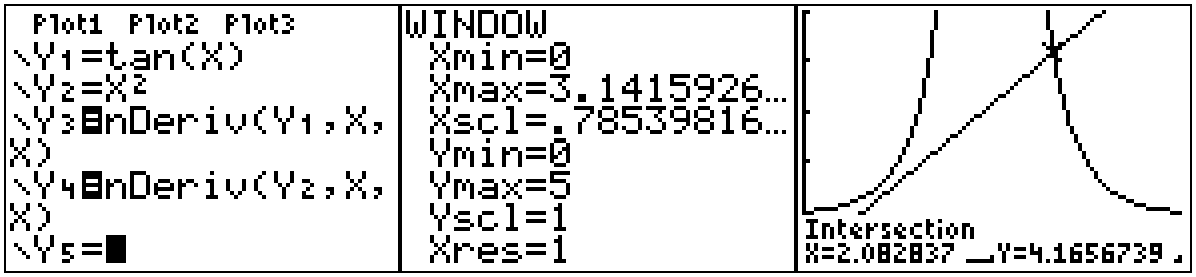
$\frac{d}{dt} a = \frac{d}{dt} \pi r^2$ $\quad \frac{dr}{dt} = 4 \frac{\text{in.}}{\text{sec}}$

$\frac{da}{dt} = 2\pi r \frac{dr}{dt}$ $\quad r = 12$

$\frac{dA}{dt} = 2\pi(12 \text{ inches})\left(4 \frac{\text{in}}{\text{sec}}\right) = 96\pi \frac{\text{in}^2}{\text{sec}}$

13. On the interval $[0, \pi]$, where do the graphs of $f(x) = \tan x$ and $g(x) = x^2$ have parallel tangent lines?

IF PARALLEL, Then SAME Slope
SO Find OUT WHERE $f'(x) = g'(x)$ on $[0, \pi]$



Hence, at $x \approx 2.082837$

14.
The position $s(t)$ of a particle is measured every 10 seconds and is provided in the table below.

t [in seconds]	0	10	20	30	40	50
$s(t)$ [in feet]	0	12	15	17	30	60

(A) Estimate the instantaneous velocity of the particle at $t = 25$ seconds. Include units.

$$v(25) \approx \frac{s(30) - s(20)}{30 - 20} = \frac{17 - 15}{10} = \frac{1}{5}\ \frac{ft}{sec}$$

(B) Find the average velocity for the time interval [0, 50]

AV VEL ON [0,50]

$$= \frac{s(50) - s(0)}{50 - 0} = \frac{60 - 0}{50} = \frac{6}{5}\ \frac{ft}{sec}$$

15. Find the following:

(A) $\frac{d}{dx}[7\cos^3(\pi x)]$

$$= 7\frac{d}{dx}[\cos(\pi x)]^3$$

$u = \cos \pi x$

$\frac{du}{dx} = -\pi \sin \pi x$

$$= 7\frac{d}{dx}[u^3]$$

$$= 21u^2\frac{du}{dx}$$

$$= -21\pi[\cos(\pi x)]^2[\sin(\pi x)]$$

(B) $\frac{d}{dt}\left[\frac{\pi}{3}r^2h\right]$

$$= \frac{\pi}{3}\left[\left(2r\frac{dr}{dt}\right)(h) + \left(\frac{dh}{dt}\right)(r^2)\right]$$

$$= \frac{2\pi}{3}rh\frac{dr}{dt} + \frac{\pi}{3}r^2\frac{dh}{dt}$$

16.

2008 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

6. Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\frac{dy}{dx} = \frac{-(x+1)}{2(y^3+1)}$.

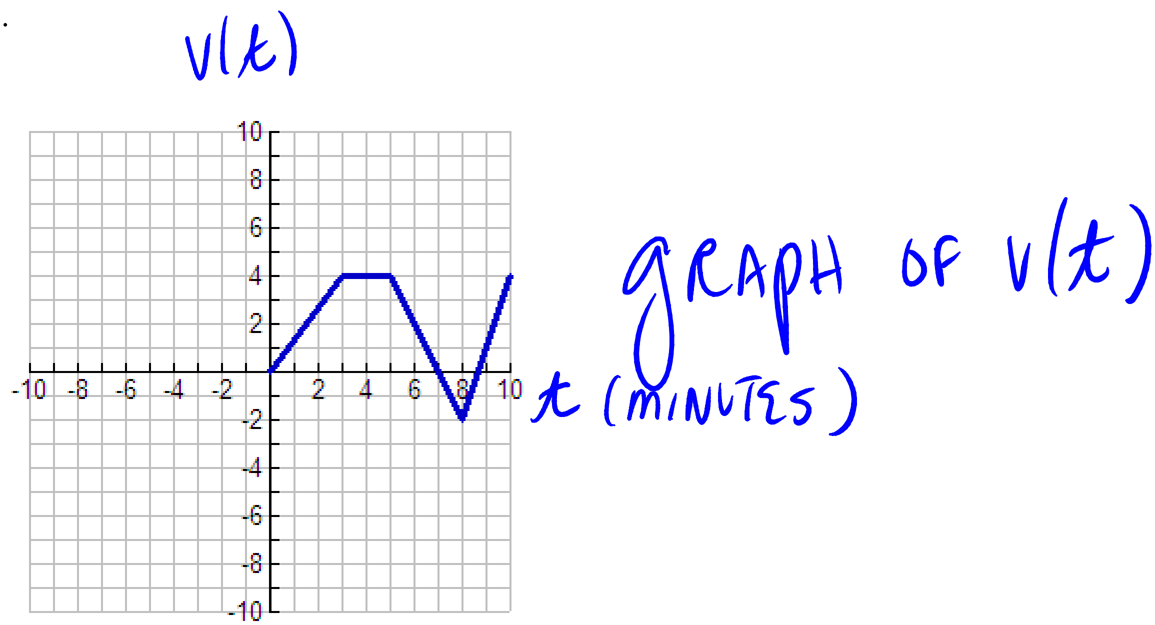
(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.

(a) $2x + 2 + 4y^3 \dfrac{dy}{dx} + 4\dfrac{dy}{dx} = 0$ $\left(4y^3 + 4\right)\dfrac{dy}{dx} = -2x - 2$ $\dfrac{dy}{dx} = \dfrac{-2(x+1)}{4\left(y^3+1\right)} = \dfrac{-(x+1)}{2\left(y^3+1\right)}$	$2 : \begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{verification} \end{cases}$
(b) $\left.\dfrac{dy}{dx}\right\|_{(-2,\,1)} = \dfrac{-(-2+1)}{2(1+1)} = \dfrac{1}{4}$ Tangent line: $y = 1 + \dfrac{1}{4}(x+2)$	$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line equation} \end{cases}$
(c) Vertical tangent lines occur at points on the curve where $y^3 + 1 = 0$ (or $y = -1$) and $x \neq -1$. On the curve, $y = -1$ implies that $x^2 + 2x + 1 - 4 = 5$, so $x = -4$ or $x = 2$. Vertical tangent lines occur at the points $(-4, -1)$ and $(2, -1)$.	$3 : \begin{cases} 1 : y = -1 \\ 1 : \text{substitutes } y = -1 \text{ into the} \\ \qquad \text{equation of the curve} \\ 1 : \text{answer} \end{cases}$
(d) Horizontal tangents occur at points on the curve where $x = -1$ and $y \neq -1$. The curve crosses the x-axis where $y = 0$. $(-1)^2 + 2(-1) + 0^4 + 4 \cdot 0 \neq 5$ No, the curve cannot have a horizontal tangent where it crosses the x-axis.	$2 : \begin{cases} 1 : \text{works with } x = -1 \text{ or } y = 0 \\ 1 : \text{answer with reason} \end{cases}$

17.



The graph above is the velocity graph of a particle moving along the x-axis.

(A) When is the particle at rest? Justify.

The particle is at rest when v(t) = 0. Hence, at t = 0, t = 7, and t ≈ 8.5, the particle is at rest.

(B) When does the particle change direction? Justify.

At t = 7, v(t) changes from positive to negative values and at t ≈ 8.5, v(t) changes from negative to positive values. Hence, the particle changes direction at t = 7 and at t ≈ 8.5.

(C) What is the acceleration at time, t = 2.

$a(t) = v'(t)$ So,

$$a(2) = v'(2) = \frac{v(3) - v(0)}{3 - 0}$$

$$a(2) = \frac{4}{3}$$

Be sure to remember how to do anything that we have done on any projects, puzzles, or quizzes for this chapter. [I want to save paper!] ***Revised on 30 September 2009***