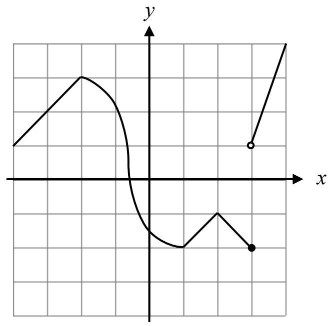
Calculus
Chapter 3 Review Sheet

1. The function for $f(x)$ is graphed below. There is a vertical tangent line when $x = -\frac{1}{2}$
Where is $f(x)$ NOT differentiable? Why?



2. If $f(x)$ has a derivative at $x = 2$, tell whether or not each of the following must be true?

a) $\lim_{x\to 2} f(x)$ exists
b) $f'(2)$ exists
c) $f''(2)$ exists.
d) $f(x)$ is continuous at $x = 2$.
e) $\lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$ exists.
f) $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$ exists.

3. Use the *alternative definition of the derivative* to find each of the following:

a) $f'(x)$ if $f(x) = \frac{3}{x}$.

b) $\frac{dy}{dx}$ if $f(x) = 34$

c) $y'(1)$ if $y = 3x^2 + 5x$

4. Find the following limits: (if you are spending a lot of time on this, you aren't "seeing" the point)

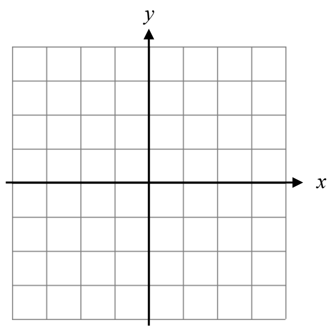
a) $\lim_{h\to 0} \frac{\tan\left(\frac{\pi}{4}+h\right)-\tan\left(\frac{\pi}{4}\right)}{h}$

b) $\lim_{h\to 0} \frac{\sqrt{4+h}-2}{h}$

5. If $f(x) = \begin{cases} 2ax^2 + b & x \geq 1 \\ -3x+4 & x < 1 \end{cases}$, find a and b so that f is both continuous and differentiable.
(Be sure to use definitions to justify your work)

6. Use the function $f(x)=\begin{cases} x & \text{if } 0\le x\le 1 \\ 2-x & \text{if } 1<x\le 2 \end{cases}$.

a) Graph the function



b) Is f continuous at $x = 1$? Explain.

c) Is f differentiable at $x = 1$? Explain.

7. Suppose $f(x)=\begin{cases} 2x-3 & \text{if } -1\le x<0 \\ x-3 & \text{if } 0\le x\le 4 \end{cases}$

a) Where is the function differentiable?

b) Where is the function continuous but not differentiable?

c) Where is the function neither continuous nor differentiable?

8. Find the equation of the *tangent line* to the curve $y = 2\sin x\cos x$ at $x = \frac{\pi}{2}$.

9. Find the derivative of each of the following:

a) $y = x^{-5} - \frac{x^3}{8} + \frac{1}{4}\sqrt[3]{x}$

b) $y = 3\sec x\csc x$

c) $y = \frac{2x+1}{3x-4}$

10. Using values from the chart, estimate a value for $f'(3)$ and explain its meaning in the context of this problem. Show how you arrived at your answer.

x = minutes	$f(x)$ = $
1	4
2	6
3	9
4	11

11. [No Calculator] Suppose $x(t) = t^2 - 8t + 12$ is a position of a particle moving along the x axis at time t.

a) Find the average velocity for the first 3 seconds.

b) Find the velocity at $t = 4$ seconds.

c) When is the object stopped?

d) When is the acceleration of the object 0?

e) When does the object change direction?

f) When does the object slow down?

g) When is the object moving left?

12. [Calculator] A particle is moving along the x-axis and its position function at time t is given by the equation $s(t) = \sqrt{t}\cos t$, where $0 \le t \le 2\pi$.

a) Find the zeros of $s(t)$. What does this tell you?

b) Find the velocity of the object at any time t.

c) Find the zeros of $v(t)$. What does this tell you?

d) Find the acceleration of the object at any time t.

e) Find the zeros of $a(t)$.

d) When does the object change direction? Justify your response.

f) When does the object speed up? When does it slow down? Justify your response.

13. On Earth, if you shoot a small object 64 feet straight up in the air, the object will be $h(t) = 64t - 16t^2$ feet above your head at t seconds after launching.

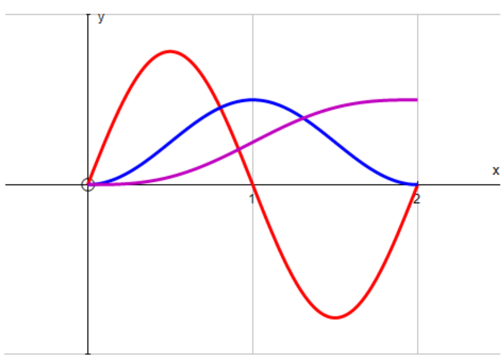
a) Find $\frac{dh}{dt}$ and $\frac{d^2h}{dt^2}$ and explain what you have found.

b) Is the object speeding up or slowing down at $t = 1$ second? Justify your response.

14. If a hemispherical bowl of radius 10 in. is filled with water to a depth of x in., the volume of water is given by $V = \pi\left[10 - \frac{x}{3}\right]x^2$.

Find the rate of increase of the volume. Indicate units of measure.

15. The following graphs show the distance traveled, velocity, and the acceleration for each second of a 2-minute automobile trip. Which graph shows distance? Velocity? Acceleration? Explain your choice.



16. Suppose that a function f and its first derivative have the following values at $x = 0$ and $x = 1$.

x	$f(x)$	$f'(x)$
0	9	–2
1	–3	1/5

Find an expression for the derivative of the following combinations, then find the derivative at the indicated point.

a) $\sqrt{x}f(x)$ at $x = 1$

b) $\frac{f(x)}{2+\cos x}$ at $x = 0$

17. Suppose that functions f and g and their first derivatives have the following values at $x = -1$ and $x = 0$.

Find an expression for the derivative of the following combinations , then find the derivative at the indicated point.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
–1	0	–1	2	1
0	–1	–3	–2	4

a) $3f(x) - g(x)$ at $x = -1$

b) $\dfrac{f(x)}{g(x)+2}$ at $x = 0$

c) $f(x) \cdot 4g(x)$ at $x = 0$

18. Use your calculator to find the derivative of the following functions at the indicated point. Label each correctly.

a) $y = \sqrt{3x-8}$ when $x = 2$

b) $f(x) = \cos(3x)$ when $x = \pi/2$

c) $P(x) = \dfrac{x^2 \sin x + \tan x}{x+7}$ when $x = 0$.

19. [Calculator] The number of the zombies in a small town is modeled by the equation $Z(t) = \dfrac{800}{1+e^{-.05t}}$, where t is the number of days since the initial outbreak and $Z(t)$ is the total number of zombies.

a) Calculate the number of zombies initially.

b) What is the rate of change in the number of zombies on day 5? Indicate units of measure.