AP Calculus
2.1 Worksheet Day 1

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. The only way to guarantee the existence of a limit is to algebraically prove it. Describe the different ways you can *investigate* the existence of a limit.

2. Using words, explain what is meant by the expression $\lim_{q \to c} f(q) = T$.

3. How do you find the average speed of an object?

4. Suppose an object moves along the x-axis with it's position function given by $x(t) = 5t^2 + 7t$, where t is measured in seconds.

   a) What is the average speed from $t = 2$ to $t = 4$ seconds?

   b) How fast is the object moving at exactly $t = 4$ seconds?

5. An rover on another planet drops an object off a cliff. The object falls $y = gt^2$ m in t sec, where g is a constant. Five seconds after the object was dropped it lands 30 m below.

   a) Find the value of g.

   b) Find the average speed for the fall.

   c) With what speed did the rock hit the bottom?

6. Assume $\lim_{x \to b} f(x) = 7$ and $\lim_{x \to b} g(x) = -3$.

   a) $\lim_{x \to b}(f(x) + g(x)) =$

   b) $\lim_{x \to b}(f(x) \cdot g(x)) =$

   c) $\lim_{x \to b} 4g(x) =$

   d) $\lim_{x \to b}\left(\frac{f(x)}{g(x)}\right) =$

7. When asked to evaluate the limit of a function, what should be done first?

8. Evaluate the following limits by using direct substitution.

a) $\lim_{x \to 7} \sec\left(\frac{\pi x}{6}\right)$

b) $\lim_{x \to 4} \sqrt[3]{x+4}$

c) $\lim_{x \to \frac{1}{2}} 3x^2(2x-1)$

d) $\lim_{y \to 2} \frac{y^2+5y+6}{y+2}$

e) $\lim_{x \to -2} (x-6)^{\frac{2}{3}}$

f) $\lim_{x \to 2} \sqrt{x+3}$

9. Explain why you <u>cannot</u> use direct substitution to determine each of the following limits.

a) $\lim_{x \to -2} \sqrt{x-2}$

b) $\lim_{x \to 0} \frac{1}{x^2}$

c) $\lim_{x \to 0} \frac{(4+x)^2-16}{x}$

10. If a limit does not exist, there are 3 possible reasons why. List all three possible reasons why a limit may not exist.

11. Find each limit, or explain why the limit does not exist.

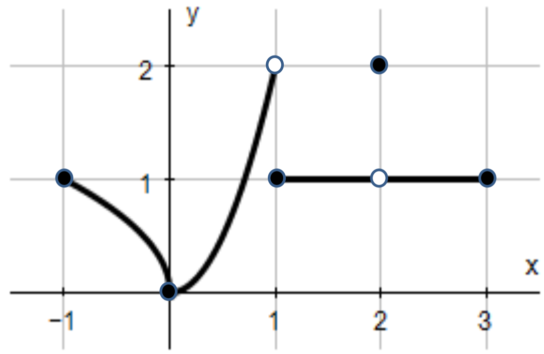
a) $\lim_{x \to 2} f(x)$, if $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln x & \text{for } 2 < x \le 4 \end{cases}$

b) $\lim_{x \to 2^+} f(x)$, if $f(x) = \begin{cases} 3x+1 & , x < 2 \\ \frac{5}{x+1} & , x \ge 2 \end{cases}$

c) $\lim_{x \to 1} \frac{x^2-4}{x-1}$

d) $\lim_{x \to 2} \frac{x+1}{x^2-4}$

12. Determine whether each statement about the graph below is True or False.



a) $\lim_{x\to -1^+} f(x) = 1$

b) $\lim_{x\to 2} f(x)$ does not exist

c) $\lim_{x\to 2} f(x) = 2$

d) $\lim_{x\to 1^-} f(x) = 2$

e) $\lim_{x\to 1^+} f(x) = 1$

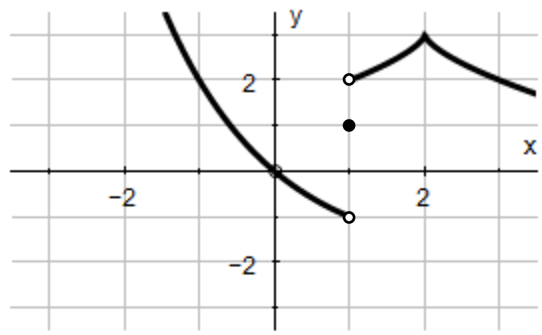
f) $\lim_{x\to 1} f(x)$ does not exist

g) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$

h) $\lim_{x\to c} f(x)$ exists at every c in (–1, 1)

i) $\lim_{x\to c} f(x)$ exists at every c in (1, 3)

13. Use the graph of $f(x)$ to estimate the limits and value of the function, or explain why the limit does not exist.



a) $\lim_{x\to 1^+} f(x)$

b) $\lim_{x\to 1^-} f(x)$

c) $\lim_{x\to 1} f(x)$

d) $f(1)$

e) $\lim_{x\to 2^+} f(x)$

f) $\lim_{x\to 2^-} f(x)$

g) $\lim_{x\to 2} f(x)$

h) $f(2)$

14. For each of the following functions, (i) draw the graph, (ii) determine $\lim_{x\to c^+} f(x)$ and $\lim_{x\to c^-} f(x)$, and (iii) explain what the value of $\lim_{x\to c} f(x)$ is or explain why it doesn't exist.

a) $c = 2,\ f(x) = \begin{cases} 6 - x, & \text{if } x < 2 \\ 4, & \text{if } x = 2 \\ \frac{x}{2} + 3, & \text{if } x > 2 \end{cases}$

b) $c = -1,\ f(x) = \begin{cases} 1 - x^2, & \text{if } x \neq -1 \\ 3, & \text{if } x = -1 \end{cases}$

15. Suppose $f(x)=\begin{cases}\sqrt{1-x^2}, & \text{if } 0\le x<1\\ 3, & \text{if } 1\le x<2\\ 1, & \text{if } x=2\end{cases}$. Draw a graph of $f(x)$, then answer the following questions.

a) At what points c in the domain of f does $\lim_{x\to c} f(x)$ exist?

b) At what point(s) c does only the left-hand limit exist?

c) At what point(s) c does only the right-hand limit exist?

16. A water balloon dropped from the roof of a small building falls $y=4.9t^2$ m in t sec. Suppose you wanted to know the speed of the water balloon at exactly $t = 2$ seconds. Originally, we used values of t really close to 2 and found the average rate of change between them. Let's try something a little different …

a) Instead of using a numeric value "close" to 2, what would be the average speed of the balloon between $t = 2$ and $t = 2 + h$? (Simplify the expression as much as you can)

b) To find the speed of the balloon at $t = 2$, it is tempting to simply plug in $h = 0$, however, this yields $\frac{0}{0}$, which is an "indeterminate form". We CAN however, evaluate your simplified expression from part *a* using limit as $h \to 0$. Evaluate this limit.

c) Now find and compare the speed of the balloon at $t = 2$ like you did earlier in question #4.

AP Calculus
2.1 Worksheet Day 2

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. When evaluating limits, what does it mean if direct substitution gives you $\frac{7}{0}$?

2. When evaluating limits, what does it mean if direct substitution gives you $\frac{0}{0}$?

3. What are the methods (options) for dealing with the result $\frac{0}{0}$?

4. Evaluate the following limits algebraically.

a) $\lim\limits_{x \to 3} \dfrac{x^2 - x - 6}{x - 3}$

b) $\lim\limits_{x \to 0} \dfrac{\frac{1}{2+x} - \frac{1}{2}}{x}$

c) $\lim\limits_{x \to 0} \dfrac{\sqrt{2x+1} - 1}{x}$

d) $\lim\limits_{x \to 4} \dfrac{\sqrt{x+5} - 3}{x - 4}$

e) $\lim\limits_{x \to 1} \dfrac{x - 1}{x^2 - 1}$

f) $\lim\limits_{x \to 0} \dfrac{(4+x)^2 - 16}{x}$

g) $\lim\limits_{t \to 2} \dfrac{t^2 - 3t + 2}{t^2 - 4}$

h) $\lim\limits_{x \to 0} \dfrac{(2+x)^3 - 8}{x}$

One of the limits you should know is $\lim\limits_{x\to 0}\frac{\sin x}{x} = $ ______ . This limit ONLY works when the denominator matches the inside of the sine function. If they do not match, you cannot change the inside of a sine function without a trig identity. Your goal will be to correctly show the algebra in order to use this limit.

5. Evaluate each of the following limits analytically. *Be sure to show ALL steps in your evaluation*.

a) $\lim\limits_{x\to 0}\frac{\sin x}{5x}$

b) $\lim\limits_{x\to 0}\frac{\sin 5x}{x}$

c) $\lim\limits_{x\to \pi/4}\frac{\sin\left(x-\pi/4\right)}{x-\pi/4}$

d) $\lim\limits_{x\to 0}\frac{3\sin 4x}{\sin 3x}$

6. Evaluate each of the following by combining properties of limits and your algebra skills.

a) $\lim\limits_{x\to 0}\frac{x+\sin x}{x}$

b) $\lim\limits_{x\to 0}\frac{\tan x}{x}$

c) $\lim\limits_{x\to 0}\frac{\sin x}{2x^2-x}$

d) $\lim\limits_{x\to 0}\frac{\sin^2 x}{x}$

7. Consider $\lim\limits_{x\to 0}\frac{f(x)-f(0)}{x^2} =$

a) If you use direct substitution, what result do you get?

b) Evaluate the limit if $f(x)=2x^2+1$.

8. If $a\neq 0$, then $\lim\limits_{x\to a}\frac{x^2-a^2}{x^4-a^4} =$

9. Evaluate the following limits analytically (all mixed up):

a) $\lim_{x\to 0}\frac{\frac{3}{4+x}-\frac{3}{4}}{x}$

b) $\lim_{x\to 0}\frac{5x^3+8x^2}{3x^4-16x^2}$

c) $\lim_{x\to 3}\frac{\sqrt{x+1}}{x-4}$

d) $\lim_{x\to 0}\frac{x^2-3x}{x}$

e) $\lim_{x\to 1}\frac{x}{x^2-x}$

f) $\lim_{x\to 0}\frac{\sin 2x}{x}$

g) $\lim_{x\to 0}\frac{\sin 7x}{3x}$

h) $\lim_{x\to 4}\frac{x^2-5x+4}{x^2-2x-8}$

12. Evaluate $\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$.

♬: *h* is going to 0 … not *x* … so treat this as if *h* is the variable … your final answer will have a *x* in it.

13. Suppose $g(x)=\begin{cases}2-x, & \text{if } x\le 1\\ \frac{x}{2}+1, & \text{if } x>1\end{cases}$.

a) $\lim_{x\to 1^-} g(x)=$

b) $\lim_{x\to 1^+} g(x)=$

c) $\lim_{x\to 1} g(x)=$

d) $g(1)=$

AP Calculus
2.2 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Answer the following questions:

   a) How do you find horizontal asymptotes?

   b) Which of the parent functions have horizontal asymptotes? List the function(s) and asymptote(s)

   c) How do you find vertical asymptotes?

   d) Which of the parent functions have vertical asymptotes? List the function(s) and asymptote(s)

   e) When must you look for oblique (slanted) asymptotes? How do you find them?

2. For each of the following, find (i) $\lim\limits_{x\to\infty} f(x)$ and (ii) $\lim\limits_{x\to-\infty} f(x)$. Then (iii) identify all horizontal asymptotes, if any.

   a) $f(x)=\dfrac{x-2}{2x^2+3x-5}$

   b) $f(x)=\dfrac{4x^3-2x+1}{x^2-2x+1}$

   c) $f(x)=\dfrac{3x^2-x+5}{x^2-4}$

   d) $f(x)=\dfrac{e^{-x}}{x}$

   e) $f(x)=\dfrac{|x|}{x}$

   f) $f(x)=\dfrac{\sin x}{2x^2+x}$

3. One of the functions in 2a – 2c has a slanted (oblique) asymptote. Explain why, and then find the asymptote.

4. For each of the following, (i) find the vertical asymptotes of the graph of $f(x)$ and (ii) describe the behavior of the graph of $f(x)$ to the left and right of each asymptote.

a) $f(x)=\dfrac{1}{x-3}$ b) $f(x)=\dfrac{1}{x^2-4}$ c) $f(x)=\dfrac{1-x}{2x^2-5x-3}$

5. Find the limit of $g(x)$ as (i) $x\to\infty$, (ii) $x\to-\infty$, (iii) $x\to 0^-$, and (iv) $x\to 0^+$

a) $g(x)=\begin{cases}\frac{1}{x} & \text{if } x<0\\ \frac{2x-3}{x+1} & \text{if } x\geq 0\end{cases}$ b) $g(x)=\begin{cases}\frac{3x}{x+1} & \text{if } x\leq 0\\ \frac{1}{x^2} & \text{if } x>0\end{cases}$

6. Sketch a **function** that satisfies the stated conditions. Include any asymptotes.

$\lim_{x\to 1} f(x)=2$ $\lim_{x\to 5^-} f(x)=\infty$

$\lim_{x\to 5^+} f(x)=\infty$ $\lim_{x\to\infty} f(x)=-1$

$\lim_{x\to -\infty} f(x)=0$ $\lim_{x\to -2^-} f(x)=\infty$

$\lim_{x\to -2^+} f(x)=-\infty$

7. Sketch a **function** that satisfies the stated conditions. Include any asymptotes.

$\lim_{x\to 2} f(x)=-1$ $\lim_{x\to 4^+} f(x)=-\infty$

$\lim_{x\to 4^-} f(x)=\infty$ $\lim_{x\to\infty} f(x)=\infty$

$\lim_{x\to -\infty} f(x)=2$

8. Explain why there is no value L for which $\lim_{x\to\infty} \sin x = L$.

9. Let $f(x) = \dfrac{\cos x}{x}$.

a) Find the domain and range of f.

b) Is f even, odd, or neither? Justify your response.

c) Find $\lim_{x\to\infty} f(x)$. Give a reason for your answer.

10. If k is a positive integer, then $\lim_{x\to\infty} \dfrac{x^k}{e^x} =$? Explain your answer.
[Try letting $k = 2$ … what about $k = 10$? … what about $k = 1000$?]

11. ***Investigate*** using tables and graphs to determine the value of each limit: $\lim_{x\to\infty} \dfrac{3x-2}{\sqrt{2x^2+1}}$ and $\lim_{x\to-\infty} \dfrac{3x-2}{\sqrt{2x^2+1}}$

12. Evaluate each of the following limits using all methods learned from this chapter.

a) $\lim_{x\to\infty} \left(\dfrac{2}{x}+1\right)\left(\dfrac{5x^2-1}{x^2}\right)$

b) $\lim_{n\to\infty} \dfrac{3n^3-5n}{n^3-2n^2+1} =$

c) $\lim_{x\to\infty} \left(5-\dfrac{2}{x^2}\right) =$

d) $\lim_{x\to\pi/2} \sec x$

e) $\lim_{x \to \infty} e^{-x} \cos x$

f) $\lim_{x \to 7/2^{+}} \operatorname{int}(2x-1)$

g) $\lim_{x \to \infty} \dfrac{\cos\left(\frac{1}{x}\right)}{1+\frac{1}{x}}$

h) $\lim_{x \to \infty} \dfrac{4n^3}{n^2+10000n} =$

i) $\lim_{x \to 0} \dfrac{\sin 2x}{4x}$

j) $\lim_{x \to 0} \dfrac{\frac{1}{2+x}-\frac{1}{2}}{x}$

k) $\lim_{x \to \infty} \dfrac{x\sin x + 2\sin x}{2x^2}$

l) $\lim_{x \to -2} \dfrac{x^2+1}{3x^2-2x+5}$

AP Calculus
2.3 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

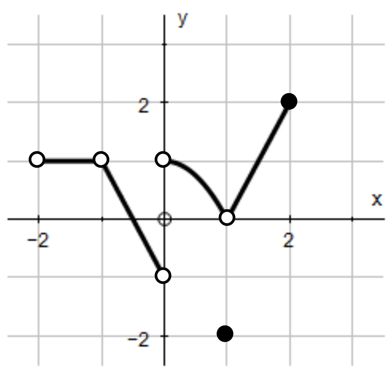
1. What is the definition of continuity (at a point)?

2. Sketch a possible graph for each function described.

   a) $f(5)$ exists, but $\lim_{x\to 5} f(x)$ does not exist.

   b) The $\lim_{x\to 5} f(x)$ exists, but $f(5)$ does not exist.

3. Use the function $g(x)$ defined and graphed below to answer the following questions.



$$g(x)=\begin{cases}1 & \text{if } -2<x<-1\\ -2x-1 & \text{if } -1<x<0\\ 1-x^2 & \text{if } 0<x<1\\ -2 & \text{if } x=1\\ 2x-2 & \text{if } 1<x\le 2\end{cases}$$

a) Does $g(1)$ exist?

b) Does $\lim_{x\to 1} g(x)$ exist?

c) Does $\lim_{x\to 1} g(x)=g(1)$?

d) Is g continuous at $x = 1$?

e) Is g defined at $x = -1$?

f) Is g continuous at $x = -1$?

g) For what values of x is g continuous?

h) What value should be assigned to $g(-1)$ to make a new (extended) function continuous at $x = -1$?

i) What value should be re-assigned to $g(1)$ to make g continuous at $x = 1$?

j) Is it possible to extend g to be continuous at $x = 0$? If so, what value should the extended function have there? If not, why not?

4. Let $f(x)=\begin{cases} x^2-1 & ;x<3 \\ 2ax & ;x\geq 3 \end{cases}$. Find a value of a so that the function f is continuous.

Using the definition of continuity, justify your response.

5. Let $f(x)=\begin{cases} 2x+3 & ;x\leq 2 \\ kx+1 & ;x>2 \end{cases}$. Find a value of k so that the function f is continuous.

Using the definition of continuity, justify your response.

6. Let $f(x)=\begin{cases} x^2-a^2x & ;x<2 \\ 4-2x^2 & ;x\geq 2 \end{cases}$. Find all values of a that make f continuous at 2.

Using the definition of continuity, justify your response.

7. If $f(x)=\begin{cases} \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} & \text{if } x\neq 2 \\ k+3 & \text{if } x=2 \end{cases}$, and if f is continuous at $x = 2$, then $k = ?$

Using the definition of continuity, justify your response.

8. If the function f is continuous for all real numbers and if $f(x)=\dfrac{x^2-4}{x+2}$, when $x\neq -2$, then $f(-2) =$

Use the definition of continuity to justify your response.

9. Let f be the function defined by the following:

$$f(x)=\begin{cases}\sin x, & x<0\\ x^2, & 0\le x<1\\ 2-x, & 1\le x<2\\ x-3, & x\ge 2\end{cases}$$

For what values of x is f NOT continuous? Use the definition of continuity to explain why.

10. Write an extended function (see questions 3h and 3i) so that the given function is continuous at the indicated point.

a) $h(x)=\dfrac{\sin(5x)}{x}$ at $x=0$

b) $k(x)=\dfrac{x-4}{\sqrt{x}-2}$ at $x=4$

11. Multiple Choice: Let f be the function given by $f(x)=\dfrac{(x-1)(x^2-4)}{x^2-a}$. For what positive values of a is f continuous for all real numbers?

A) None

B) 1 only

C) 2 only

D) 4 only

E) 1 and 4

12. Let $g(x) = \dfrac{x^2 + 5x + 6}{x^2 + 7x + 10}$.

a) Find the domain of $g(x)$.

b) Find the $\lim_{x \to c} g(x)$ for all values of c where $g(x)$ is not defined.

c) Find any horizontal asymptotes and justify your response.

d) Find any vertical asymptotes and justify your response.

e) Write an extension to the function so that $g(x)$ is continuous at $x = -2$.
Use the definition of continuity to justify your response.

13. Without using a picture, give a written explanation of why the function $f(x) = x^2 - 4x + 3$ has a zero in the interval [2, 4].

14. Without using a picture, give a written explanation of why the function $f(x) = x^2 + 2x - 3$ must equal 3 at least once in the interval [0, 2].

15. Let $h(x) = \begin{cases} 3x^2 - 4, & \text{if } x \leq 2 \\ 5 + 4x, & \text{if } x > 2 \end{cases}$.

a) What is $h(0)$?

b) What is $h(4)$?

c) On the interval [0, 4], there is no value of x such that $h(x) = 10$ even though $h(0) < 10$ and $h(4) > 10$. Explain why this result does not contradict the IVT.