AP® Exam Practice Questions for Chapter 4

1. The equation of the line is $f'(x) = -\frac{2}{5}x + 4.$

$$f(x) = \int f'(x)\,dx = \int \left(-\tfrac{2}{5}x + 4\right) dx = -\tfrac{1}{5}x^2 + 4x + C.$$

Use $f(0) = 3$ to find C.

$$3 = -\tfrac{1}{5}(0)^2 + 4(0) + C \Rightarrow C = 3$$

$$f(x) = -\tfrac{1}{5}x^2 + 4x + 3$$

$$f(10) = -\tfrac{1}{5}(10)^2 + 4(10) + 3 = 23$$

So, the answer is D.

2. $$\int_0^5 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^5 f(x)\,dx = \frac{1}{2}(3 + 5)3 + 2(-2) = 8$$

So, the answer is C.

3. Let $u = 16 - 3x^2 \Rightarrow du = -6x\,dx.$

$$\int x\sqrt{16 - 3x^2}\,dx = -\tfrac{1}{6}\int \sqrt{16 - 3x^2}(-6x)dx = -\tfrac{1}{6}\left[\tfrac{2}{3}\left(16 - 3x^2\right)^{3/2}\right] + C = -\frac{1}{9}\left(16 - 3x^2\right)^{3/2} + C$$

So, the answer is C.

4. Average value $= \dfrac{1}{b - a}\displaystyle\int_a^b f(x)\,dx$

$$12 = \frac{1}{k}\int_o^k x^3\,dx$$

$$12 = \frac{1}{k}\left[\frac{1}{4}x^4\right]_0^k$$

$$12 = \frac{1}{k}\left(\frac{1}{4}k^4 - 0\right)$$

$$12 = \frac{k^3}{4}$$

$$48 = k^3$$

$$48^{1/3} = k$$

So, the answer is D.

5. $v(t) = 4t^3 - 4t,\ 0 \le t \le 2$

$$\frac{1}{2 - 0}\int_0^2 v(t)\,dt = \frac{1}{2}\int_0^2 \left(4t^3 - 4t\right) dt = \frac{1}{2}\Big[t^4 - 2t^2\Big]_0^2 = \frac{1}{2}(16 - 8) = 4 \text{ units/sec}$$

So, the answer is A.

6. $$g(-1) = \int_4^{-1} f(t)\,dt = -\int_{-1}^4 f(t)\,dt = -\int_{-1}^0 f(t)\,dt - \int_0^3 f(t)\,dt - \int_3^4 f(t)\,dt = -\tfrac{1}{2}(-2) - \tfrac{1}{2}(4)(-2) - \tfrac{1}{2}(2) = 4$$

So, the answer is C.

7. $$\int_b^{\pi} \sin x\,dx = 0.4$$

$$\Big[-\cos x\Big]_b^{\pi} = 0.4$$

$$-\cos \pi + \cos b = 0.4$$

$$1 + \cos b = 0.4$$

$$\cos b = -0.6$$

$$b \approx 2.214$$

So, the answer is D.

8. $f'(x) = \sqrt{x^3 + 6}$

$a = 1,\ b = 5$

$$f(b) = f(a) + \int_a^b f'(x)\,dx$$

$$f(5) = f(1) + \int_1^5 \sqrt{x^3 + 6}\,dx \approx 2 + 24.672 = 26.672$$

So, the answer is D.

9. $$\frac{1}{\frac{3\pi}{2} - 0}\int_0^{3\pi/2} (x + \sin x)\,dx = \frac{2}{3\pi}\left[\frac{1}{2}x^2 - \cos x\right]_0^{3\pi/2} = \frac{2}{3\pi}\left[\left(\frac{9\pi^2}{8} - 0\right) - (0 - 1)\right] \approx 2.568$$

So, the answer is C.

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10. (a) $\int_0^{12} C'(t)\,dt = C(12) - C(0)$

$= 40 - 65$

$= -25°\text{C}$

The total temperature lost from $t = 0$ to $t = 12$ is 25°C.

3 pts: { 2 pts: answer with justification [indicates $c(12) - c(0)$] and units; 1 pt: interpretation of answer }

Note: Could simply explain as "change in temperature from time $t = 0$ to $t = 12$."

(b) $C'(4) \approx \dfrac{C(5) - C(3)}{5 - 3} = \dfrac{50 - 57}{2}$

$= -3.5$

So, when $t = 4$, the temperature of the coffee is changing about $-3.5°\text{C}$ per minute.

2 pts: completes difference quotient with units

(c) $C(t) = \int C'(t)\,dt = \int -2\cos 0.5t\,dt$

$= -2\left(\dfrac{1}{0.5}\right)\int \cos(0.5t)(0.5)\,dt$

$= -4(\sin 0.5t) + K$

$= -4\sin 0.5t + K$

Because $C(t)$ is continuous at $C = 12$, use $C(12) = 40$ to find *K*.

$-4\sin[0.5(12)] + K = 40$

$K \approx 38.8823$

So, $C(15) = -4\sin(0.5 \cdot 15) + 38.8823$

$\approx 35.130°\text{C}.$

4 pts: { 2 pts: finds antiderivative; 1 pt: uses $C(12) = 40$ to find constant of integration; 1 pt: finds $C(15)$ with units }

Note: Use more than three decimal places when representing the constant of integration (avoid premature rounding in this intermediate step) so that the final answer can be rounded to three decimal places. Perhaps store the value of the constant in your calculator for use in the subsequent computation.

Note: Round each answer to at least three decimal places to receive credit on the exam.

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11. (a) $\int_0^6 M(t)\,dt = \int_0^6 \frac{\pi}{6}\sin\frac{\pi t}{12}\,dt$

$$= 2\int_0^6 \sin\frac{\pi t}{12}\left(\frac{\pi}{12}\right)dt$$

$$= 2\left[-\cos\frac{\pi t}{12}\right]_0^6$$

$$= 2[0 - (-1)]$$

$$= 2$$

So, 2 inches of snow will melt during the 6 hour period.

3 pts:
- 1 pt: definite integral $\left[\text{writing } \int_0^6 M(t)\,dt \text{ is sufficient}\right]$
- 1 pt: answer (no work needed, use calculator)
- 1 pt: units

(b) $I(t) = S(t) - M(t) + 40$

$$= 0.006t^2 - 0.12t + 0.87 - \frac{\pi}{6}\sin\frac{\pi t}{12} + 40$$

$$= 0.006t^2 - 0.12t + 40.87 - \frac{\pi}{6}\sin\frac{\pi t}{12}$$

2 pts:
- 1 pt: includes $S(t) - M(t)$ in answer
- 1 pt: includes the initial value, 40, in answer

(c) $I'(t) = 0.012t - 0.12 - \frac{\pi}{6}\cos\frac{\pi t}{12}\left(\frac{\pi}{12}\right)$

$$I'(3) = 0.012(3) - 0.12 - \frac{\pi^2}{72}\cos\left(\frac{\pi}{4}\right)$$

$$\approx -0.181 \text{ in./h}$$

2 pts:
- 1 pt: answer [computes $I'(3)$, no work needed]
- 1 pt: units (a rate)

(d) Because $I(t)$ is decreasing over $[0, 6]$, the maximum is at $t = 0$.

$I(0) = 40.87$, so the maximum amount of snow is 40.87 inches.

2 pts:
- 1 pt: answers
- 1 pt: justification [identifies that $I(t)$ is decreasing on this interval]

Note: Round each answer to at least three decimal places to receive credit on the exam.

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12. (a) $v(t) = \sin \frac{\pi t}{4}$

The particle is moving to the right on the t-intervals $(0, 4)$ and $(8, 9)$ because $v(t) > 0$ on these intervals.

2 pts: answers with justification $\left[\text{identifies where } v(t) > 0\right]$

Note: Be sure to explicitly identify each function by name. Referring to $v(t)$ as "it," "the function," or "the graph" may not receive credit on the exam.

(b) $\int_0^9 \sin \frac{\pi t}{4}\, dt$

1 pt: definite integral

(c) $v'(t) = \frac{\pi}{4} \cos \frac{\pi t}{4}$

$$a(3) = v'(3)$$
$$= \frac{\pi}{4} \cos \frac{3\pi}{4}$$

Because $3\pi/4$ is in Quadrant II, $\cos(3\pi/4)$ is negative. So, $a(3) < 0$. Because $v(3) > 0$, the acceleration and velocity are in opposite directions. This means that the particle is slowing down.

3 pts:
- 1 pt: computes $a(t) = v'(t)$ (Chain Rule)
- 1 pt: answer for $a(3)$
- 1 pt: answer ("slowing down") with explanation

Note: $a(3)$ does *not* need to be evaluated or simplified. Leaving the answer as $\frac{\pi}{4} \cos \frac{3\pi}{4}$ or $\frac{\pi}{4}\left(-\frac{\sqrt{2}}{2}\right)$ is sufficient.

(d) $s(t) = \int v(t)\, dt$

$$= \int \sin \frac{\pi t}{4}\, dt$$
$$= \frac{4}{\pi} \int \sin \frac{\pi t}{4}\left(\frac{\pi}{4}\right) dt$$
$$= -\frac{4}{\pi} \cos \frac{\pi t}{4} + C$$

Use $s(0) = -4$ to find C.

$$-4 = -\frac{4}{\pi} \cos 0 + C$$
$$-4 + \frac{4}{\pi} = C$$

So, $s(t) = -\frac{4}{\pi} \cos \frac{\pi t}{4} + \frac{4 - 4\pi}{\pi}$.

Therefore, $s(3) = -\frac{4}{\pi} \cos \frac{3\pi}{4} + \frac{4 - 4\pi}{4}$.

3 pts:
- 1 pt: antiderivative
- 1 pt: uses initial condition to find constant of integration
- 1 pt: answer

Note: The answer does not need to be evaluated or simplified.

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13. $F(x) = \int_3^x f(t)\,dt$

(a) $F(0) = \int_3^0 f(t)\,dt$

$= -\int_0^3 f(t)\,dt$

$= -\left[\frac{1}{4}\pi(2)^2 + \frac{1}{2}(2 + 3)(1)\right]$

≈ -5.64

$F'(0) = f(0) = 3$

$F(4) = \int_3^4 f(t)\,dt$

$= \frac{1}{2}(-1)(1) = -0.5$

3 pts:
- 1 pt: answer for $F(0)$ with justification (computes signed area)
- 1 pt: answer for $F'(0)$ with justification [indicates $F'(0) = f(0)$]
- 1 pt: answer for $F(4)$ with justification (computes signed area)

(b) The graph of $F(x)$ does not have a minimum value because $F'(x) = f(x)$ does not change from negative to positive at any point.

1 pt: answer with justification [identifies that $F'(x) = f(x)$ does not change from negative to positive on this interval]

(c) Because $F'(x) = f(x)$ changes from increasing to decreasing at $x = 0$, an inflection point of the graph of $F(x)$ is $x = 0$.

2 pts: answer with reason [identifies where $F'(x) = f(x)$ changes from increasing to decreasing]

(d) Because $F'(2) = f(2) = 1$, the slope of the tangent line is 1. Use $F(2) = \int_3^2 f(t)\,dt = -\frac{1}{2}$ and $m = 1$ to write the equation of the tangent line.

$y + \frac{1}{2} = 1(x - 2)$

$y = x - \frac{5}{2}$

So, the equation of the line tangent to the graph of F at $x = 2$ is $y = x - \frac{5}{2}$.

3 pts:
- 1 pt: finds *slope* of the line tangent to the graph of F at $x = 2$ [finds $F'(2) = f(2)$]
- 1 pt: finds $F(2)$ (signed area)
- 1 pt: finds an *equation* of the line tangent to F at $x = 2$

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14. (a) $\int_0^1 f(x)\,dx = -5.5$

$F(1) - F(0) = -5.5$

$9 - F(0) = -5.5$

$F(0) = 14.5$

$\int_1^3 f(x)\,dx = -6$

$F(3) - F(1) = -6$

$F(3) - 9 = -6$

$F(3) = 3$

$\int_3^4 f(x)\,dx = 15.5$

$F(4) - F(3) = 15.5$

$F(4) - 3 = 15.5$

$F(4) = 18.5$

So, $F(0) = 14.5$ and $F(4) = 18.5$.

4 pts:
- 2 pts: finds $F(0)$ with justification [uses $\int_0^1 f(x)\,dx = -5.5$ and $F(1) = 9$]
- 2 pts: finds $F(4)$ with justification [uses $\int_1^3 f(x)\,dx = -6$ and $F(1) = 9$ to find $F(3)$, and then uses $\int_3^4 f(x)\,dx = 15.5$ and $F(3) = 3$ to find $F(4)$]

(b) Because $F(0) > 5 > F(3)$ and $F(x)$ is continuous on $[0, 3]$, there is at least one x-value in this interval where $F(x) = 5$ by the Intermediate Value Theorem. Because $F(3) < 5 < F(4)$ and $F(x)$ is continuous on $[3, 4]$, there is at least one x-value in this interval where $F(x) = 5$. So, $F(x)$ must equal 5 at least 2 times on $[0, 4]$.

3 pts: answer with reason [appeals to values of $F(0)$, $F(3)$, and $F(4)$ and the continuity of $F(x)$]

(c) Because $F'(x) = f(x) > 0$ on the interval $(3, 4)$, F is increasing on the interval $(3, 4)$.

2 pts: answer with justification [appeals to where $F'(x) = f(x) > 0$]

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