

AP[®] Exam Practice Questions for Chapter 1

$$1. \lim_{x \rightarrow \pi} \frac{\sin x}{x} = \frac{\sin \pi}{\pi} = \frac{0}{\pi} = 0$$

So, the answer is A.

$$2. \lim_{x \rightarrow -2} \frac{3x^2 + 5x + 7}{x - 4}$$

$$= \frac{3(-2)^2 + 5(-2) + 7}{(-2) - 4}$$

$$= \frac{9}{-6}$$

$$= -\frac{3}{2}$$

So, the answer is B.

3. Evaluate each statement.

I: As x approaches 3 from the left and right, the function approaches 1. So, $\lim_{x \rightarrow 3} \sqrt{x - 2} = 1$ is a true statement.

II: As x approaches 3 from the left and right, the function approaches 0. So, $\lim_{x \rightarrow 3} (6 - 2x) = 0$ is a true statement.

III: As x approaches 3 from the left, the function approaches 0 and as x approaches 3 from the right, the function approaches 1. So, the limit $\lim_{x \rightarrow 3} f(x)$ does not exist is a true statement.

Because I, II, and III are true statements, the answer is D.

8. Evaluate each statement.

A: Because $\lim_{x \rightarrow 1^-} f(x) = \infty$ and $\lim_{x \rightarrow 1^+} = -\infty$, $\lim_{x \rightarrow 1} f(x)$ does not exist. So, $\lim_{x \rightarrow 1} f(x) = \infty$ is false.

B: Because $\lim_{x \rightarrow 3^-} f(x) = 3$ and $\lim_{x \rightarrow 3^+} = 2$, $\lim_{x \rightarrow 3^-} f(x) > \lim_{x \rightarrow 3^+} f(x)$. So, $\lim_{x \rightarrow 3^-} f(x) < \lim_{x \rightarrow 3^+} f(x)$ is false.

C: Because $\lim_{x \rightarrow 3^-} f(x) = 3$ and $\lim_{x \rightarrow 3^+} f(x) = 2$, $\lim_{x \rightarrow 3} f(x)$ does not exist. So, $\lim_{x \rightarrow 3} f(x) = 1$ is false.

D: Because $\lim_{x \rightarrow 0^+} f(x) = 2$ and $\lim_{x \rightarrow 3^+} f(x) = 2$, $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 3^+} f(x)$. So, $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 3^+} f(x)$ is true.

So, the answer is D.

4. Evaluate each limit.

I: Using a graphing utility, $\lim_{x \rightarrow 1} \frac{x^3 + 1}{x - 1}$ does not exist.

II: $\lim_{x \rightarrow 0} \frac{|x|}{x} = \lim_{x \rightarrow 0} f(x)$, where $f(x) = \begin{cases} -1, & x < 0 \\ 1, & x \geq 0 \end{cases}$ does not exist because the limits on each side of $x = 0$ do not agree.

III: $\lim_{x \rightarrow 2} f(x)$, where $f(x) = \begin{cases} 3, & x \leq 2 \\ 0, & x > 2 \end{cases}$ does not exist because the limits on each side of $x = 2$ do not agree.

Because the limits of I, II, and III do not exist, the answer is D.

5. The domain of

$$f(x) = \frac{2}{\sqrt{x-1}} \text{ is } \sqrt{x-1} > 0 \Rightarrow x > 1.$$

Because f is not continuous at $x = 1$, the answer is C.

$$6. \lim_{x \rightarrow 5} [5f(x) - g(x)] = \lim_{x \rightarrow 5} 5f(x) - \lim_{x \rightarrow 5} g(x)$$

$$= 5 \lim_{x \rightarrow 5} f(x) - \lim_{x \rightarrow 5} g(x)$$

$$= 5(10) - (1) = 49$$

So, the answer is D.

7. Evaluate each statement.

I: Because $\lim_{x \rightarrow 2^-} g(x) = 1$ and $\lim_{x \rightarrow 2^+} g(x) = 1$,
 $\lim_{x \rightarrow 2} g(x) = 1$.

The statement is true.

II: $\lim_{x \rightarrow 2} g(x) = 1 \neq g(2) = 3$

The statement is false.

III: g is continuous at $x = 3$.

The statement is true.

Because I and III are true, the answer is B.

9. The function $f(x) = \frac{10}{x^4}$ increases without bound as x approaches 0 from the left and as x approaches 0 from the right. So, the limit is nonexistent. So, the answer is D.

11. (a) $s(1) = 393.1$
 $s(2) = 378.4$
 Because s is continuous on $[1, 2]$ and $s(2) < 382 < s(1)$, by the Intermediate Value Theorem there exists a time t in the open interval $(1, 2)$ such that $s(t) = 382$ meters.

- (b) $s(t) = -4.9t^2 + 398$
 $0 = -4.9t^2 + 398$
 $t \approx 9.012$

The negative solution does not make sense in the context of the problem. So, the object hits the ground after approximately 9.012 seconds.

$$\begin{aligned} \text{(c) } \lim_{t \rightarrow 3} \frac{s(t) - s(3)}{t - 3} &= \lim_{t \rightarrow 3} \frac{(-4.9t^2 + 398) - (-4.9(3)^2 + 398)}{t - 3} \\ &= \lim_{t \rightarrow 3} \frac{-4.9t^2 + 4.9(9)}{t - 3} \\ &= \lim_{t \rightarrow 3} \frac{-4.9(t^2 - 9)}{t - 3} \\ &= \lim_{t \rightarrow 3} \frac{-4.9(t - 3)(t + 3)}{t - 3} \\ &= \lim_{t \rightarrow 3} -4.9(t + 3) \\ &= -4.9(3 + 3) \\ &= -29.4 \text{ m/sec} \end{aligned}$$

10. Because $\lim_{x \rightarrow 1^-} \frac{x - 1}{\sqrt{x} - 1} = 2$ and $\lim_{x \rightarrow 1^+} \frac{x - 1}{\sqrt{x} - 1} = 2$,
 $\lim_{x \rightarrow 1} \frac{x - 1}{\sqrt{x} - 1} = 2$.

So, the answer is C.

- 4 pts: $\left\{ \begin{array}{l} 2 \text{ pts: shows } s(2) < 382 < s(1) \\ \quad \text{(places 382 in this interval)} \\ 2 \text{ pts: justification (appeals to the} \\ \quad \text{continuity of } s \text{ and/or the} \\ \quad \text{Intermediate Value Theorem)} \end{array} \right.$

Note: Merely saying “because s is differentiable” or “because s is decreasing” would not earn the justification points.

1 pt: answer with units

- 4 pts: $\left\{ \begin{array}{l} 2 \text{ pts: justification of limit by factoring} \\ 2 \text{ pts: answer with units} \end{array} \right.$

$$\begin{aligned} 12. (a) \lim_{x \rightarrow 1} [f(x) + 4] &= \lim_{x \rightarrow 1} f(x) + \lim_{x \rightarrow 1} 4 \\ &= 2 + 4 \\ &= 6 \end{aligned}$$

1 pt: answer

$$(b) \lim_{x \rightarrow 3^-} \frac{5}{g(x)} = \frac{5}{1} = 5$$

1 pt: answer

$$(c) \lim_{x \rightarrow 2} [f(x)g(x)] = (2)(0) = 0$$

2 pts: $\left\{ \begin{array}{l} 1 \text{ pt: justification (indicates the values of both } f(2) \\ \text{and } g(2)); \text{ writing } (2)(0) \text{ is sufficient} \end{array} \right.$

1 pt: answer

$$\begin{aligned} (d) \lim_{x \rightarrow 3} \frac{f(x)}{g(x) - 1} &= \lim_{x \rightarrow 3} \frac{(-2x + 6)}{(x - 2) - 1} \\ &= \lim_{x \rightarrow 3} \frac{-2x + 6}{x - 3} \\ &= \lim_{x \rightarrow 3} \frac{-2(x - 3)}{(x - 3)} \\ &= -2 \end{aligned}$$

5 pts: $\left\{ \begin{array}{l} 2 \text{ pts: finds linear equations for } f \text{ and } g \text{ on } [2, 3] \\ 2 \text{ pts: justification of limit by factoring} \\ 1 \text{ pt: answer} \end{array} \right.$

Note: The equations for f and g must both be correct to be eligible for the last three points (i.e., the limit must yield the indeterminate form $0/0$).

13. (a) Because $\lim_{x \rightarrow 2^-} f(x) = 1$ and $\lim_{x \rightarrow 2^+} f(x) = 1$, and the limits are the same, $\lim_{x \rightarrow 2} f(x) = 1$.

3 pts: $\left\{ \begin{array}{l} 2 \text{ pts: computes each one-sided limit} \\ 1 \text{ pt: equates the one-sided limits to reach a} \\ \text{conclusion about } \lim_{x \rightarrow 2} f(x) \end{array} \right.$

(b) Because $f(2) = 3$, $\lim_{x \rightarrow 2} f(x) = 1$, and $f(2) \neq \lim_{x \rightarrow 2} f(x)$, f is not continuous at $x = 2$.

4 pts: $\left\{ \begin{array}{l} 1 \text{ pt: computes } f(2) \\ 2 \text{ pts: indicates } f(2) \neq \lim_{x \rightarrow 2} f(x) \\ 1 \text{ pt: reaches correct conclusion} \end{array} \right.$

$$(c) \lim_{x \rightarrow 2} [\sin(f(x))] = \sin\left[\lim_{x \rightarrow 2} f(x)\right] = \sin 1$$

2 pts: finds limit

$$14. (a) f(x) = \frac{x^2 + 5x + 6}{2x^2 + 7x + 3} = \frac{(x+2)(x+3)}{(2x+1)(x+3)}$$

$$= \frac{x+2}{2x+1}, x \neq -3$$

$f(x)$ has discontinuities at $x = -\frac{1}{2}$

and $x = -3$.

$$(b) \lim_{x \rightarrow -3} f(x) = \lim_{x \rightarrow -3} \frac{x^2 + 5x + 6}{2x^2 + 7x + 3}$$

$$= \lim_{x \rightarrow -3} \frac{(x+2)(x+3)}{(2x+1)(x+3)}$$

$$= \lim_{x \rightarrow -3} \frac{x+2}{2x+1}$$

$$= \frac{-3+2}{2(-3)+1}$$

$$= \frac{1}{5}$$

(c) $f(x)$ has a vertical asymptote at $x = -\frac{1}{2}$.

4 pts: answers with justification (justify where denominator equals zero)

3 pts: $\begin{cases} 2 \text{ pts: justification by factoring} \\ 1 \text{ pt: answer} \end{cases}$

2 pts: answer

Note: Including $x = -3$ in the answer (a removable discontinuity) would lose one of the answer points.

15. (a) Because $T(x)$ is continuous on $[0, 10)$,

$$\lim_{x \rightarrow 4} T(x) = T(4) = 172.$$

$$\begin{aligned} \text{(b)} \quad \frac{T(8) - T(3)}{8 - 3} &= \frac{164 - 174}{8 - 3} \\ &= \frac{-10}{5} \\ &= -2 \end{aligned}$$

The average rate of change is -2°F per minute.

- (c) $T(x)$ is continuous and when $x = 6$,

$$T(x) > 166.5^\circ \text{ and when } x = 8,$$

$$T(x) < 166.5^\circ.$$

So, the shortest interval is $(6, 8)$.

- (d) Because $T(x)$ is continuous, the average rate of change for $6 \leq x \leq 9$ is

$$\begin{aligned} \frac{T(9) - T(6)}{9 - 6} &= \frac{162 - 168}{9 - 6} \\ &= \frac{-6}{3} \\ &= -2. \end{aligned}$$

So, the tangent line at $x = 8$ has a slope of about -2 .

2 pts: $\begin{cases} 1 \text{ pt: answer} \\ 1 \text{ pt: justification (appeals to the continuity of } T) \end{cases}$

2 pts: $\begin{cases} 1 \text{ pt: justification (evidence of a difference quotient)} \\ 1 \text{ pt: answer with units} \end{cases}$

3 pts: $\begin{cases} 1 \text{ pt: shows } T(8) < 166.5 < T(6) \\ \quad \text{(places } 166.5 \text{ in this interval)} \\ 1 \text{ pt: answer (an open or closed interval would} \\ \quad \text{generally be accepted here)} \\ 1 \text{ pt: justification (appeals to the continuity of } T \\ \quad \text{or the Intermediate Value Theorem)} \end{cases}$

Note: Merely stating “because T is differentiable” or “because T is decreasing” would not earn the justification point.

2 pts: $\begin{cases} 1 \text{ pt: justification (evidence of a difference quotient} \\ \quad \text{on } [6, 9]) \\ 1 \text{ pt: answer} \end{cases}$

$$16. (a) \quad f(x) = ax^2 + x - b \qquad f(x) = ax + b$$

$$f(2) = a(2)^2 + (2) - b \qquad f(2) = a(2) + b$$

$$= 4a + 2 - b \qquad = 2a + b$$

f is continuous at $x = 2$ when $4a + 2 - b = 2a + b$.

$$f(x) = ax + b \qquad f(x) = 2ax - 7$$

$$f(5) = a(5) + b \qquad f(5) = 2a(5) - 7$$

$$= 5a + b \qquad = 10a - 7$$

f is continuous at $x = 5$ when $5a + b = 10a - 7$.

$$4a + 2 - b = 2a + b \Rightarrow 2a - 2b = -2$$

$$5a + b = 10a - 7 \Rightarrow -5a + b = -7$$

Multiply both sides of the second equation by 2.

$$2a - 2b = -2$$

$$\frac{-10a + 2b = -14}{-8a} = -16$$

$$a = 2$$

When $a = 2$, $b = 5(2) - 7 = 10 - 7 = 3$.

So, f is continuous when $a = 2$ and $b = 3$.

$$(b) \quad \lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} (2x + 3) = 2(3) + 3 = 9$$

$$(c) \quad \lim_{x \rightarrow 1} g(x) = \lim_{x \rightarrow 1} \frac{f(x)}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{2x^2 + x - 3}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{(2x + 3)(x - 1)}{x - 1}$$

$$= \lim_{x \rightarrow 1} (2x + 3)$$

$$= 2(1) + 3$$

$$= 5$$

5 pts: $\left\{ \begin{array}{l} 3 \text{ pts: equates } 4a + 2 - b \text{ and } 2a + b \\ \quad \text{(equates the one-sided limits at } x = 2) \\ \quad \text{and equates } 5a + b \text{ and } 10a - 7 \\ \quad \text{(equates the one-sided limits at } x = 5) \\ 2 \text{ pts: answers (solves the system of equations} \\ \quad \text{to determine } a \text{ and } b) \end{array} \right.$

2 pts: answer from using values of a and b from part (a) and the correct piece of f

Note: Incorrect values of a and b may be imported from part (a) as long as a is nonzero (i.e., an answer consistent with values for part (a) can generally earn these two points).

2 pts: $\left\{ \begin{array}{l} 1 \text{ pt: justification by factoring} \\ 1 \text{ pt: answer} \end{array} \right.$

Note: To be eligible for these points, the limit must yield the indeterminate form $\frac{0}{0}$ using the values of a and b from part (a) (i.e., incorrect values of a and b generally cannot be imported here).