AP REVIEW SESSION 7

Applications of Definite Integrals

- Integrals as Net Change
- Area between Curves
  - Washer Method
  - Disk Method
- Volumes

No Calculator

2008

17. What is the area of the region enclosed by the graphs of $f(x) = x - 2x^2$ and $g(x) = -5x$?

(A) $\frac{7}{3}$ (B) $\frac{16}{3}$ (C) $\frac{20}{3}$ (D) 9 (E) 36

Calculator Allowed

2003

82. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)\,dt$

(B) $\int_0^8 r(t)\,dt$

(C) $\int_0^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$

(E) $\int_0^{2.667} r'(t)\,dt$

84. A pizza, heated to a temperature of 350 degrees Fahrenheit (°F), is taken out of an oven and placed in a 75°F room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

(A) 112°F (B) 119°F (C) 147°F (D) 238°F (E) 335°F

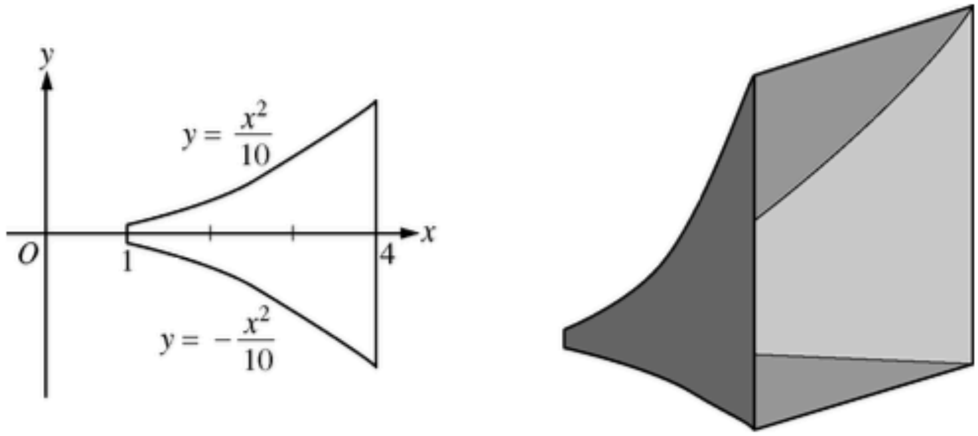
86. The base of a solid is the region in the first quadrant bounded by the y-axis, the graph of $y = \tan^{-1} x$, the horizontal line $y = 3$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) 2.561 (B) 6.612 (C) 8.046 (D) 8.755 (E) 20.773

2008

79. A spherical tank contains 81.637 gallons of water at time $t = 0$ minutes. For the next 6 minutes, water flows out of the tank at the rate of $9\sin\left(\sqrt{t+1}\right)$ gallons per minute. How many gallons of water are in the tank at the end of the 6 minutes?

(A) 36.606 (B) 45.031 (C) 68.858 (D) 77.355 (E) 126.668

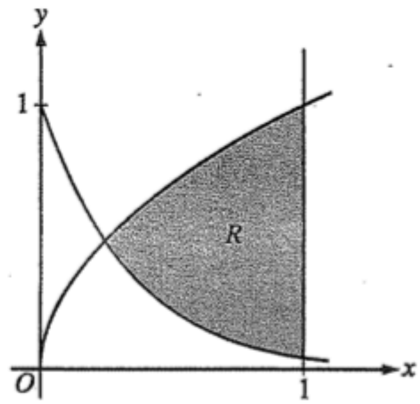


85. The base of a loudspeaker is determined by the two curves $y = \frac{x^2}{10}$ and $y = -\frac{x^2}{10}$ for $1 \le x \le 4$, as shown in the figure above. For this loudspeaker, the cross sections perpendicular to the x-axis are squares. What is the volume of the loudspeaker, in cubic units?

(A) 2.046 (B) 4.092 (C) 4.200 (D) 8.184 (E) 25.711

FRQ – Calculator Allowed

2003



1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

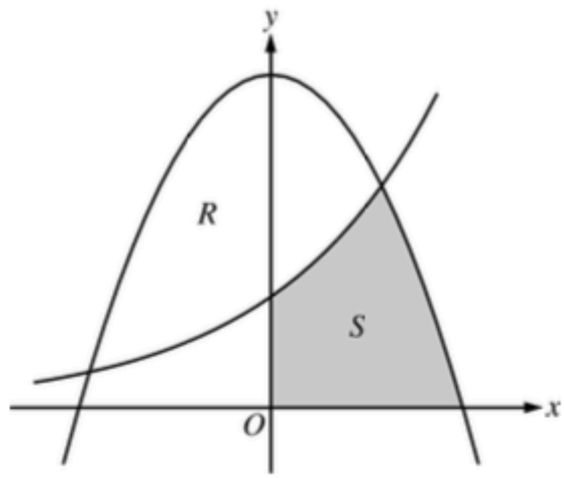
2008

1. The rate at which raw sewage enters a treatment tank is given by $E(t) = 850 + 715\cos\left(\frac{\pi t^2}{9}\right)$ gallons per hour for $0 \le t \le 4$ hours. Treated sewage is removed from the tank at the constant rate of 645 gallons per hour. The treatment tank is empty at time $t = 0$.

   (a) How many gallons of sewage enter the treatment tank during the time interval $0 \le t \le 4$? Round your answer to the nearest gallon.

   (b) For $0 \le t \le 4$, at what time t is the amount of sewage in the treatment tank greatest? To the nearest gallon, what is the maximum amount of sewage in the tank? Justify your answers.

   (c) For $0 \le t \le 4$, the cost of treating the raw sewage that enters the tank at time t is $(0.15 - 0.02t)$ dollars per gallon. To the nearest dollar, what is the total cost of treating all the sewage that enters the tank during the time interval $0 \le t \le 4$?



2. Let R and S in the figure above be defined as follows: R is the region in the first and second quadrants bounded by the graphs of $y = 3 - x^2$ and $y = 2^x$. S is the shaded region in the first quadrant bounded by the two graphs, the x-axis, and the y-axis.

   (a) Find the area of S.

   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -1$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with one leg across the base of the solid. Write, but do not evaluate, an integral expression that gives the volume of the solid.