AP REVIEW SESSION 6

Differential Equations & Modeling

- Derivatives of Inverse Functions
- Differential Equations
- Exponential Growth & Decay
- Slope Fields
- Definite/Indefinite Integration with U-substitution

No-Calc

2003

2. $\int_0^1 e^{-4x}\,dx =$

(A) $\frac{-e^{-4}}{4}$ (B) $-4e^{-4}$ (C) $e^{-4}-1$ (D) $\frac{1}{4}-\frac{e^{-4}}{4}$ (E) $4-4e^{-4}$

8. $\int x^2\cos\left(x^3\right)dx =$

(A) $-\frac{1}{3}\sin\left(x^3\right)+C$

(B) $\frac{1}{3}\sin\left(x^3\right)+C$

(C) $-\frac{x^3}{3}\sin\left(x^3\right)+C$

(D) $\frac{x^3}{3}\sin\left(x^3\right)+C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right)+C$

11. Using the substitution $u = 2x + 1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

(A) $\frac{1}{2}\int_{-1/2}^{1/2} \sqrt{u}\,du$ (B) $\frac{1}{2}\int_0^2 \sqrt{u}\,du$ (C) $\frac{1}{2}\int_1^5 \sqrt{u}\,du$ (D) $\int_0^2 \sqrt{u}\,du$ (E) $\int_1^5 \sqrt{u}\,du$

12. The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

(A) $V(t) = k\sqrt{t}$

(B) $V(t) = k\sqrt{V}$

(C) $\frac{dV}{dt} = k\sqrt{t}$

(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$

(E) $\frac{dV}{dt} = k\sqrt{V}$

19. A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point (1, 2) ?

(A) $y = 5x - 3$

(B) $y = x^2 + 1$

(C) $y = x^2 + 3x$

(D) $y = x^2 + 3x - 2$

(E) $y = 2x^2 + 3x - 3$

27. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

2008

1. $\int \cos(3x)\,dx =$

(A) $-3\sin(3x) + C$

(B) $-\frac{1}{3}\sin(3x) + C$

(C) $\frac{1}{3}\sin(3x) + C$

(D) $\sin(3x) + C$

(E) $3\sin(3x) + C$

7. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx =$

(A) $2e^{\sqrt{x}} + C$

(B) $\frac{1}{2}e^{\sqrt{x}} + C$

(C) $e^{\sqrt{x}} + C$

(D) $2\sqrt{x}\,e^{\sqrt{x}} + C$

(E) $\frac{1}{2}\frac{e^{\sqrt{x}}}{\sqrt{x}} + C$

13. $\int \left(x^3+1\right)^2 \, dx =$

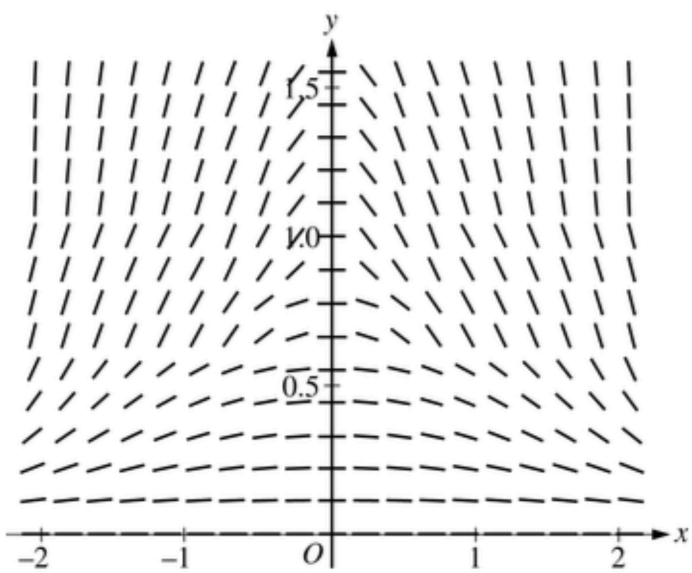
(A) $\frac{1}{7}x^7 + x + C$

(B) $\frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$

(C) $6x^2\left(x^3+1\right) + C$

(D) $\frac{1}{3}\left(x^3+1\right)^3 + C$

(E) $\dfrac{\left(x^3+1\right)^3}{9x^2} + C$



15. The slope field for a certain differential equation is shown above. Which of the following could be a solution to the differential equation with the initial condition $y(0) = 1$?

(A) $y = \cos x$

(B) $y = 1 - x^2$

(C) $y = e^x$

(D) $y = \sqrt{1 - x^2}$

(E) $y = \dfrac{1}{1 + x^2}$

28. If $y = \sin^{-1}(5x)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{1+25x^2}$

(B) $\frac{5}{1+25x^2}$

(C) $\frac{-5}{\sqrt{1-25x^2}}$

(D) $\frac{1}{\sqrt{1-25x^2}}$

(E) $\frac{5}{\sqrt{1-25x^2}}$

Calculator Allowed

2008

90. The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$. If $f(3) = 8$ and $f'(3) = 9$, what are the values of $g(8)$ and $g'(8)$?

(A) $g(8) = \frac{1}{3}$ and $g'(8) = -\frac{1}{9}$

(B) $g(8) = \frac{1}{3}$ and $g'(8) = \frac{1}{9}$

(C) $g(8) = 3$ and $g'(8) = -9$

(D) $g(8) = 3$ and $g'(8) = -\frac{1}{9}$

(E) $g(8) = 3$ and $g'(8) = \frac{1}{9}$

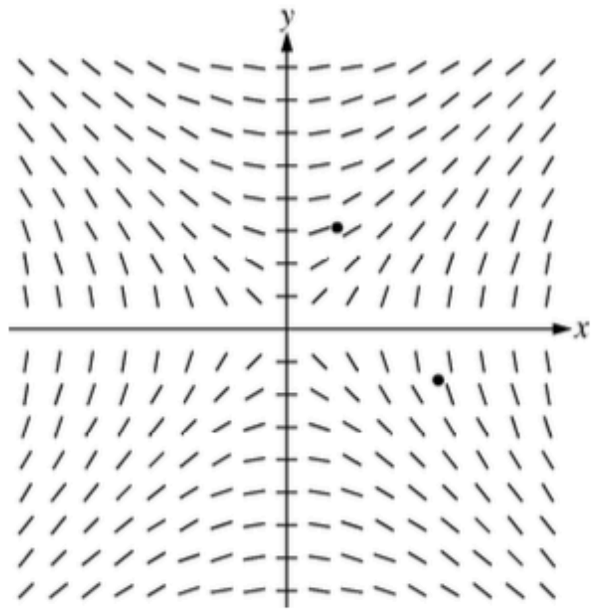
FRQ – No Calculator

2008

5. Consider the differential equation $\frac{dy}{dx} = \frac{x}{y}$, where $y \neq 0$.

(a) The slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(3, -1)$, and sketch the solution curve that passes through the point $(1, 2)$.

(Note: The points $(3, -1)$ and $(1, 2)$ are indicated in the figure.)



(b) Write an equation for the line tangent to the solution curve that passes through the point $(1, 2)$.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(3) = -1$, and state its domain.