Name: _____ Date: _____

Period: _____ AP Calc AB

Mellina/Hommen

Midterm AP Multiple Choice Review

The following questions are to be completed without a calculator

1. $\int \left(5e^{2x} + \frac{1}{x}\right) dx =$

(A) $\frac{5}{2}e^{2x} + \frac{2}{x^2} + C$

(B) $\frac{5}{2}e^{2x} + \ln|x| + C$

(C) $5e^{2x} + \frac{2}{x^2} + C$

(D) $5e^{2x} + \ln|x| + C$

(E) $10e^{2x} - \frac{1}{x^2} + C$

2. If $f(x) = \sqrt{x} + \frac{3}{\sqrt{x}}$, then $f'(4) =$

(A) $\frac{1}{16}$ (B) $\frac{5}{16}$ (C) 1 (D) $\frac{7}{2}$ (E) $\frac{49}{4}$

x	0	25	30	50
$f(x)$	4	6	8	12

4. The values of a continuous function f for selected values of x are given in the table above. What is the value of the left Riemann sum approximation to $\int_0^{50} f(x)dx$ using the subintervals $[0, 25]$, $[25, 30]$, and $[30, 50]$?

(A) 290 (B) 360 (C) 380 (D) 390 (E) 430

$$f(x) = \begin{cases} x^2 \sin(\pi x) & \text{for } x < 2 \\ x^2 + cx - 18 & \text{for } x \geq 2 \end{cases}$$

5. Let f be the function defined above, where c is a constant. For what value of c, if any, is f continuous at $x = 2$?

   (A) 2 (B) 7 (C) 9 (D) $4\pi - 4$ (E) There is no such value of c.

6. Which of the following is an antiderivative of $3\sec^2 x + 2$?

   (A) $3\tan x$ (B) $3\tan x + 2x$ (C) $3\sec x + 2x$ (D) $\sec^3 x + 2x$ (E) $6\sec^2 x \tan x$

7. If $f(x) = x^2 - 4$ and g is a differentiable function of x, what is the derivative of $f(g(x))$?

   (A) $2g(x)$ (B) $2g'(x)$ (C) $2xg'(x)$ (D) $2g(x)g'(x)$ (E) $2g(x) - 4$

9. If $f''(x) = x(x+2)^2$, then the graph of f is concave up for

   (A) $x < 0$

   (B) $x > 0$

   (C) $-2 < x < 0$

   (D) $x < -2$ and $x > 0$

   (E) all real numbers

10. If $y = \sin x \cos x$, then at $x = \frac{\pi}{3}$, $\frac{dy}{dx} =$

(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 1

11. $\lim_{x \to -3} \frac{x^2 - 9}{x^2 - 2x - 15}$ is

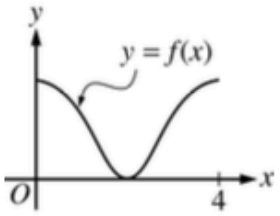
(A) 0 (B) $\frac{3}{5}$ (C) $\frac{3}{4}$ (D) 1 (E) nonexistent

12. What is the average rate of change of $y = \cos(2x)$ on the interval $\left[0, \frac{\pi}{2}\right]$?

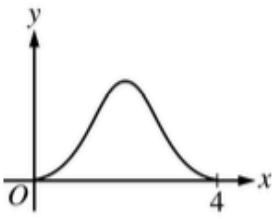
(A) $-\frac{4}{\pi}$ (B) -1 (C) 0 (D) $\frac{\sqrt{2}}{2}$ (E) $\frac{4}{\pi}$

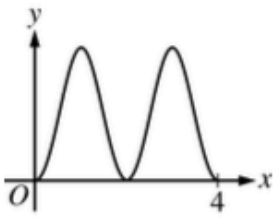
13. If $y^3 + y = x^2$, then $\frac{dy}{dx} =$

(A) 0 (B) $\frac{x}{2}$ (C) $\frac{2x}{3y^2}$ (D) $2x - 3y^2$ (E) $\frac{2x}{1 + 3y^2}$



14. The graph of $y = f(x)$ on the closed interval $[0, 4]$ is shown above. Which of the following could be the graph of $y = f'(x)$?

(A)


(B)


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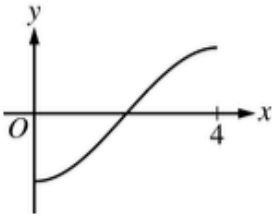

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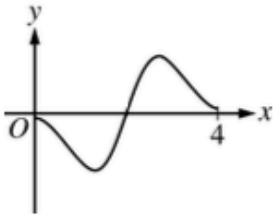

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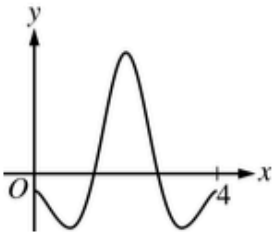


$$f(x) = \begin{cases} 3x - 2 & \text{if } x < 1 \\ \ln(3x - 2) & \text{if } x \geq 1 \end{cases}$$

15. Let f be the function defined above. Which of the following statements about f are true?

I. $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x)$

II. $\lim\limits_{x \to 1^-} f'(x) = \lim\limits_{x \to 1^+} f'(x)$

III. f is differentiable at $x = 1$.

(A) None

(B) I only

(C) II only

(D) II and III only

(E) I, II, and III

16. The function f is defined by $f(x) = 2x^3 - 4x^2 + 1$. The application of the Mean Value Theorem to f on the interval $1 \le x \le 3$ guarantees the existence of a value c, where $1 < c < 3$, such that $f'(c) =$

(A) 0 (B) 9 (C) 10 (D) 14 (E) 16

17. The velocity v, in meters per second, of a certain type of wave is given by $v(h) = 3\sqrt{h}$, where h is the depth, in meters, of the water through which the wave moves. What is the rate of change, in meters per second per meter, of the velocity of the wave with respect to the depth of the water, when the depth is 2 meters?

(A) $-\frac{3}{4\sqrt{2}}$ (B) $-\frac{3}{8\sqrt{2}}$ (C) $\frac{3}{2\sqrt{2}}$ (D) $\frac{3}{\sqrt{2}}$ (E) $4\sqrt{2}$

18. If $\frac{dy}{dt} = -10e^{-t/2}$ and $y(0) = 20$, what is the value of $y(6)$?

(A) $20e^{-6}$ (B) $20e^{-3}$ (C) $20e^{-2}$ (D) $10e^{-3}$ (E) $5e^{-3}$

19. Let f be the function with derivative defined by $f'(x) = x^3 - 4x$. At which of the following values of x does the graph of f have a point of inflection?

(A) 0 (B) $\frac{2}{3}$ (C) $\frac{2}{\sqrt{3}}$ (D) $\frac{4}{3}$ (E) 2

20. Let f be the function given by $f(x) = \dfrac{(x-4)(2x-3)}{(x-1)^2}$. If the line $y = b$ is a horizontal asymptote to the graph of f, then $b =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

22. Let f be the function given by $f(x) = \dfrac{kx}{x^2+1}$, where k is a constant. For what values of k, if any, is f strictly decreasing on the interval $(-1, 1)$?

(A) $k < 0$

(B) $k = 0$

(C) $k > 0$

(D) $k > 1$ only

(E) There are no such values of k.

23. Which of the following is an equation for the line tangent to the graph of $y = 3 - \int_{-1}^{x} e^{-t^3}\,dt$ at the point where $x = -1$?

(A) $y - 3 = -3e(x+1)$

(B) $y - 3 = -e(x+1)$

(C) $y - 3 = 0$

(D) $y - 3 = \dfrac{1}{e}(x+1)$

(E) $y - 3 = 3e(x+1)$

25. $\lim\limits_{h\to 0} \dfrac{\sin\left(\frac{\pi}{3}+h\right) - \sin\left(\frac{\pi}{3}\right)}{h}$ is

(A) 0 (B) $\dfrac{1}{2}$ (C) 1 (D) $\dfrac{\sqrt{3}}{2}$ (E) nonexistent

26. An object moves along a straight line so that at any time $t \geq 0$ its velocity is given by $v(t) = 2\cos(3t)$. What is the distance traveled by the object from $t = 0$ to the first time that it stops?

(A) 0 (B) $\frac{\pi}{6}$ (C) $\frac{2}{3}$ (D) $\frac{\pi}{3}$ (E) $\frac{4}{3}$

28. The top of a 15-foot-long ladder rests against a vertical wall with the bottom of the ladder on level ground, as shown above. The ladder is sliding down the wall at a constant rate of 2 feet per second. At what rate, in radians per second, is the acute angle between the bottom of the ladder and the ground changing at the instant the bottom of the ladder is 9 feet from the base of the wall?

(A) $-\frac{2}{9}$ (B) $-\frac{1}{6}$ (C) $-\frac{2}{25}$ (D) $\frac{2}{25}$ (E) $\frac{1}{9}$

76. The function $P(t)$ models the population of the world, in billions of people, where t is the number of years since January 1, 2010. Which of the following is the best interpretation of the statement $P'(1) = 0.076$?

(A) On February 1, 2010, the population of the world was increasing at a rate of 0.076 billion people per year.

(B) On January 1, 2011, the population of the world was increasing at a rate of 0.076 billion people per year.

(C) On January 1, 2011, the population of the world was 0.076 billion people.

(D) From January 1, 2010 to January 1, 2011, the population of the world was increasing at an average rate of 0.076 billion people per year.

(E) When the population of the world was 1 billion people, the population of the world was increasing at a rate of 0.076 billion people per year.

x	0	2	4	6	8	10
$f(x)$	5	7	8	0	–15	–20

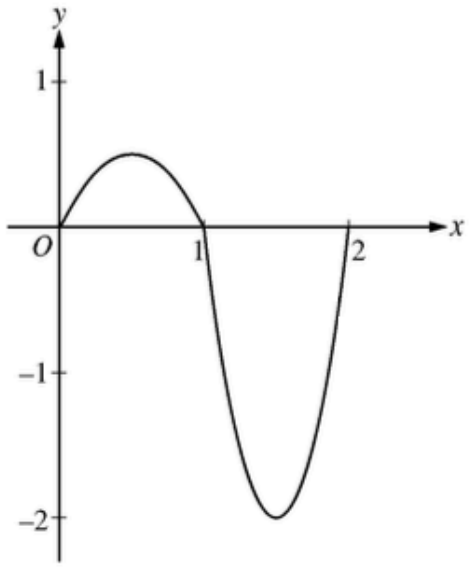
77. Let f be a differentiable function with selected values given in the table above. What is the average rate of change of f over the closed interval $0 \le x \le 10$?

(A) -6 (B) $-\frac{5}{2}$ (C) -2 (D) $-\frac{2}{5}$ (E) $\frac{2}{5}$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	3	4	2	π

79. The table above gives values of the differentiable functions f and g and their derivatives at $x = 0$. If $h(x) = \frac{f(x)}{g(x)}$, what is the value of $h'(0)$?

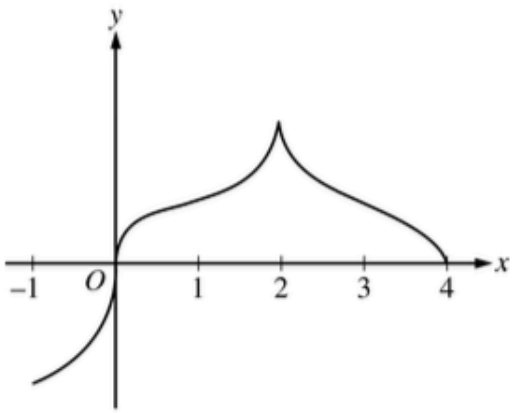
(A) $\frac{8-3\pi}{4}$ (B) $\frac{3\pi-8}{4}$ (C) $\frac{4}{\pi}$ (D) $\frac{2-3\pi}{2}$ (E) $\frac{8+3\pi}{4}$



Graph of f'

80. The figure above shows the graph of f', the derivative of a function f, for $0 \le x \le 2$. What is the value of x at which the absolute minimum of f occurs?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$ (E) 2



Graph of f'

82. The graph of f', the derivative of f, is shown above. The line tangent to the graph of f' at $x = 0$ is vertical, and f' is not differentiable at $x = 2$. Which of the following statements is true?

(A) f' does not exist at $x = 2$.

(B) f is decreasing on the interval $(2, 4)$.

(C) The graph of f has a point of inflection at $x = 2$.

(D) The graph of f has a point of inflection at $x = 0$.

(E) f has a local maximum at $x = 0$.

84. The function f is continuous on the closed interval $[1, 7]$. If $\int_1^7 f(x)\,dx = 42$ and $\int_7^3 f(x)\,dx = -32$, then $\int_1^3 2f(x)\,dx =$

(A) -148 (B) 10 (C) 12 (D) 20 (E) 148

85. Let $y = f(x)$ define a twice-differentiable function and let $y = t(x)$ be the line tangent to the graph of f at $x = 2$. If $t(x) \geq f(x)$ for all real x, which of the following must be true?

(A) $f(2) \geq 0$

(B) $f'(2) \geq 0$

(C) $f'(2) \leq 0$

(D) $f''(2) \geq 0$

(E) $f''(2) \leq 0$

86. The vertical line $x = 2$ is an asymptote for the graph of the function f. Which of the following statements must be false?

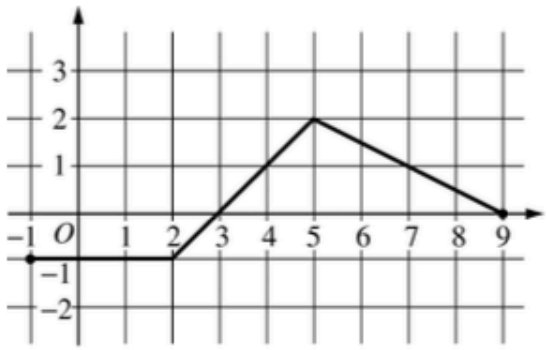
(A) $\lim_{x \to 2} f(x) = 0$

(B) $\lim_{x \to 2} f(x) = -\infty$

(C) $\lim_{x \to 2} f(x) = \infty$

(D) $\lim_{x \to \infty} f(x) = 2$

(E) $\lim_{x \to \infty} f(x) = \infty$



Graph of f

87. The graph of the piecewise linear function f is shown above. Let h be the function given by $h(x) = \int_{-1}^{x} f(t)\,dt$. On which of the following intervals is h increasing?

(A) $[-1, 3]$

(B) $[0, 5]$

(C) $[2, 5]$ only

(D) $[2, 9]$

(E) $[3, 9]$ only

89. If $\lim_{x \to a} f(x) = f(a)$, then which of the following statements about f must be true?

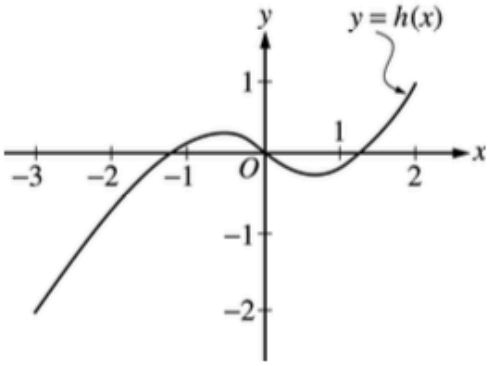
(A) f is continuous at $x = a$.

(B) f is differentiable at $x = a$.

(C) For all values of x, $f(x) = f(a)$.

(D) The line $y = f(a)$ is tangent to the graph of f at $x = a$.

(E) The line $x = a$ is a vertical asymptote of the graph of f.



92. The graph of the function h is shown in the figure above. Of the following, which has the greatest value?

(A) Average value of h over $[-3,2]$

(B) Average rate of change of h over $[-3,2]$

(C) $\int_{-3}^{2} h(x)\,dx$

(D) $\int_{-3}^{0} h(x)\,dx$

(E) $h'(0)$

Question 1: B
Question 2: A
Question 3: E
Question 4: A
Question 5: B
Question 6: B
Question 7: D
Question 8: C
Question 9: B
Question 10: A
Question 11: C
Question 12: A
Question 13: E
Question 14: D
Question 15: C
Question 16: C
Question 17: C
Question 18: B
Question 19: C
Question 20: C
Question 21: B
Question 22: A
Question 23: B
Question 24: D
Question 25: B
Question 26: C
Question 27: B
Question 28: A
Question 76: B
Question 77: B
Question 78: D
Question 79: A
Question 80: E
Question 81: B
Question 82: C
Question 83: B
Question 84: D
Question 85: E
Question 86: A
Question 87: E
Question 88: A
Question 89: A
Question 90: D
Question 91: D
Question 92: B