Sequences and Series for AP Calculus BC

Things to Know

- Know how to identify types of series:
    - Geometric
    - Harmonic
    - Alternating
    - P-series
- Determine convergence and divergence of series using various tests:
    - nth term test for divergence
    - Geometric Series
    - p-series
    - Alternating Series
    - Integral Test
    - Ratio Test
    - Direct Comparison Test
    - Limit Comparison Test
- Classify series as absolutely convergent, conditionally convergent, or divergent.

SUMMARY OF TESTS FOR SERIES

Test	Series	Condition(s) of Convergence	Condition(s) of Divergence	Comment
nth-Term	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	This test cannot be used to show convergence.
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\|r\| < 1$	$\|r\| \geq 1$	Sum: $S = \frac{a}{1-r}$
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$0 < p \leq 1$	
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1}a_n$	$0 < a_{n+1} \leq a_n$ and $\lim_{n\to\infty} a_n = 0$		Remainder: $\|R_N\| \leq a_{N+1}$
Integral (f is continuous, positive, and decreasing)	$\sum_{n=1}^{\infty} a_n$, $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	Remainder: $0 < R_N < \int_N^{\infty} f(x)\,dx$

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Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$ or $= \infty$	Test is inconclusive if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1.$
Direct Comparison ($a_n, b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \le b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \le a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Limit Comparison ($a_n, b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	

Most of the above tests are assessed in the multiple-choice portion of the test.

- Know the Maclaurin series for e^x, $\sin x$, $\cos x$, *and* $\frac{1}{1-x}$ and be prepared to find composition of functions by replacing x with a function of x.
  - $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots$ $\quad -\infty < x < \infty$
  - $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \ldots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \ldots$ $\quad -\infty < x < \infty$
  - $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \ldots + \frac{(-1)^n x^{2n}}{(2n)!} + \ldots$ $\quad -\infty < x < \infty$
  - $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \ldots + x^n + \ldots$ $\quad -1 < x < 1$
  - $\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 + \ldots + (-1)^n x^n + \ldots$ $\quad -1 < x < 1$
- Use and analyze Taylor and Maclaurin series, including differentiation, antidifferentiation, and the formation of a new series from a known series centered at the same value. Be prepared to do generate new series through composition of functions. Be able to generate power series for a given function at x=c.

  Maclaurin Series: $\frac{f(0)}{0!} + \frac{f'(0)x^1}{1!} + \frac{f''(0)x^2}{2!} + \frac{f^{(3)}(0)x^3}{3!} + \ldots + \frac{f^{(n)}(0)x^n}{n!} + \ldots$

  Taylor Series: $\frac{f(c)}{0!} + \frac{f'(c)(x-c)^1}{1!} + \frac{f''(c)(x-c)^2}{2!} + \frac{f^{(3)}(c)(x-c)^3}{3!} + \ldots + \frac{f^{(n)}(c)(x-c)^n}{n!} + \ldots$
- Understand and use intervals of convergence and error analysis (Alternating Series and LaGrange error bounds).
- Sum of convergent infinite geometric series; $s = \frac{a}{1-r}$ *provided* $|r| < 1$, *a* is initial term.
- Radius of convergence remains the same for $f(x)$, $f'(x)$, $f''(x)$, $\int (f(x))dx$. Must check convergence at end points to determine IOC if not geometric.

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You may anticipate 1 free response question out of 6 on this topic along with approximately 8 multiple-choice questions out of 45.

Convergence tests are assessed on both the free-response and multiple-choice sections.

Taylor and Maclaurin series are generally tested in both multiple-choice and free-response sections of the test.

Look for and recognize the harmonic series (divergent) and the alternating harmonic series (convergent) as these are used often for comparison purposes.