Name _____ Date _____

9.8 – 9.9 Textbook Practice

Section 9.8

Finding the Interval of Convergence **In Exercises 15–38, find the interval of convergence of the power series. (Be sure to include a check for convergence at the endpoints of the interval.)**

15. $\sum_{n=0}^{\infty} \left(\frac{x}{4}\right)^n$

17. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$

19. $\sum_{n=0}^{\infty} \frac{x^{5n}}{n!}$

21. $\sum_{n=0}^{\infty} (2n)! \left(\frac{x}{3}\right)^n$

23. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{6^n}$

25. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-4)^n}{n9^n}$

27. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1}$

29. $\sum_{n=1}^{\infty} \frac{(x-3)^{n-1}}{3^{n-1}}$

31. $\sum_{n=1}^{\infty} \frac{n}{n+1}(-2x)^{n-1}$

33. $\sum_{n=0}^{\infty} \frac{x^{3n+1}}{(3n+1)!}$

Finding Intervals of Convergence **In Exercises 49–52, find the intervals of convergence of (a) $f(x)$, (b) $f'(x)$, (c) $f''(x)$, and (d) $\int f(x)\,dx$. (Be sure to include a check for convergence at the endpoints of the intervals.)**

49. $f(x) = \sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n$

51. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1}$

Section 9.9

Finding a Geometric Power Series **In Exercises 3–6, find a geometric power series for the function, centered at 0, (a) by the technique shown in Examples 1 and 2 and (b) by long division.**

3. $f(x) = \dfrac{1}{4 - x}$

5. $f(x) = \dfrac{4}{3 + x}$

Finding a Power Series **In Exercises 7–18, find a power series for the function, centered at c, and determine the interval of convergence.**

7. $f(x) = \dfrac{1}{6 - x}, \quad c = 1$

9. $f(x) = \dfrac{1}{1 - 3x}, \quad c = 0$

11. $g(x) = \dfrac{5}{2x - 3}, \quad c = -3$

13. $f(x) = \dfrac{2}{5x + 4}, \quad c = -1$

15. $g(x) = \dfrac{4x}{x^2 + 2x - 3}, \quad c = 0$

16. $g(x) = \dfrac{3x - 8}{3x^2 + 5x - 2}, \quad c = 0$

Using a Power Series **In Exercises 19–28, use the power series**

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n, \quad |x| < 1$$

to find a power series for the function, centered at 0, and determine the interval of convergence.

21. $f(x) = -\dfrac{1}{(x+1)^2} = \dfrac{d}{dx}\left[\dfrac{1}{x+1}\right]$

25. $g(x) = \dfrac{1}{x^2+1}$

Answers:

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3. $R = 5$ **5.** 0 **7.** 2 **9.** $R = 1$ **11.** $R = \frac{1}{4}$

13. $R = \infty$ **15.** $(-4, 4)$ **17.** $(-1, 1]$ **19.** $(-\infty, \infty)$

21. $x = 0$ **23.** $(-6, 6)$ **25.** $(-5, 13]$ **27.** $(0, 2]$

29. $(0, 6)$ **31.** $\left(-\frac{1}{2}, \frac{1}{2}\right)$ **33.** $(-\infty, \infty)$ **35.** $(-1, 1)$

37. $x = 3$ **39.** $R = c$ **41.** $(-k, k)$ **43.** $(-1, 1)$

49. (a) $(-3, 3)$ (b) $(-3, 3)$ (c) $(-3, 3)$ (d) $[-3, 3)$

51. (a) $(0, 2]$ (b) $(0, 2)$ (c) $(0, 2)$ (d) $[0, 2]$

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3. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{4^{n+1}}$ **5.** $\displaystyle\sum_{n=0}^{\infty} \frac{4}{3}\left(\frac{-x}{3}\right)^n$

7. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{5}\right)\left(\frac{x-1}{5}\right)^n$ $(-4, 6)$

9. $\displaystyle\sum_{n=0}^{\infty} (3x)^n$ $\left(-\frac{1}{3}, \frac{1}{3}\right)$

11. $\displaystyle -\frac{5}{9}\sum_{n=0}^{\infty}\left[\frac{2}{9}(x+3)\right]^n$ $\left(-\frac{15}{2}, \frac{3}{2}\right)$

13. $\displaystyle -2\sum_{n=0}^{\infty}[5(x+1)]^n$ $\left(-\frac{6}{5}, -\frac{4}{5}\right)$

15. $\displaystyle\sum_{n=0}^{\infty}\left[\frac{1}{(-3)^n} - 1\right]x^n$ $(-1, 1)$

17. $\displaystyle\sum_{n=0}^{\infty} x^n[1 + (-1)^n] = 2\sum_{n=0}^{\infty} x^{2n}$ $(-1, 1)$

19. $\displaystyle 2\sum_{n=0}^{\infty} x^{2n}$ $(-1, 1)$

21. $\displaystyle\sum_{n=1}^{\infty} n(-1)^n x^{n-1}$ $(-1, 1)$

23. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$ $(-1, 1]$

25. $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{2n}$ $(-1, 1)$

27. $\displaystyle\sum_{n=0}^{\infty} (-1)^n (2x)^{2n}$ $\left(-\frac{1}{2}, \frac{1}{2}\right)$