# Taylor and Maclaurin Series – WKST 2

**I. Find the first four nonzero terms and the general term for the Taylor series for *f* about x=0.**

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| 1. $f(x)=\dfrac{\cos(3x)}{x}$ | 2. $f(x)=x^2e^{-x}$ | 3. $f(x)=\cos^2 x$ (first four terms only) | 4. $f(x)=\sin\left(x^2\right)$ |
| 5. $f(x)=\int_0^x\left(e^{-t^2}-1\right)dt$ | 6. $f(x)=\dfrac{\sin x}{x}$ | 7. $f(x)=\dfrac{e^{x/2}-1}{x}$ | |

**II. Use a power series to find the indicated limits.**

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| 8. $\lim_{x\to 0}\cos^2 x$ (use your series from #3) | 9. $\lim_{x\to 0}\dfrac{\sin x}{x}$ (use your series from #6) |

**III. Use a power series to approximate the integral value with an error less than 0.001.**

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| 10. $\int_0^1\sin\left(x^2\right)dx$ (use your series from #4) | 11. $\int_0^1 x^2e^{-x}\,dx$ (use your series from #2) |

**IV. Find the indicated if** $f(x)=1-x+x^2-x^3+\ldots+(-1)^n x^n+\ldots$

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| 12. $f'(0)$ | 13. $f^{(4)}(0)$ | 14. $f^{(9)}(0)$ |

**V. The function *f* has a Taylor series about x=3 that converges to $f(x)$ for all *x* in the interval of convergence. The *n*th derivative of *f* at x=3 is given by** $f^{(n)}(3)=\dfrac{(-1)^n n!}{5^n n^2}$ **for** $n\ge 1$, and $f(3)=0$.

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| 15. Find the first four nonzero terms and general term for the Taylor series about x=3. | 16. Find the radius of convergence of the Taylor series for *f* about x=3. Show the work that leads to your answer. |

**ANSWERS**

1. $f(x)=\frac{1}{x}-\frac{3^2x}{2!}+\frac{3^4x^3}{4!}-\frac{3^6x^5}{6!}+\ldots+\frac{(-1)^n3^{2n}x^{2n-1}}{(2n)!}+\ldots$

2. $f(x)=x^2-x^3+\frac{x^4}{2!}-\frac{x^5}{3!}+\ldots+\frac{(-1)^nx^{n+2}}{n!}+\ldots$

3. $f(x)=1-\frac{2x}{2\cdot2!}+\frac{(2x)^4}{2\cdot4!}-\frac{(2x)^6}{2\cdot6!}+\ldots$

4. $f(x)=x^2-\frac{x^6}{3!}+\frac{x^{10}}{5!}-\frac{x^{14}}{7!}+\ldots+\frac{(-1)^nx^{4n+2}}{(2n+1)!}+\ldots$

5. $f(x)=-\frac{x^3}{3}+\frac{x^5}{5\cdot2!}-\frac{x^7}{7\cdot4!}+\frac{x^9}{9\cdot6!}-\ldots+\frac{(-1)^{n+1}x^{2n+3}}{(2n+3)(2n)!}+\ldots$

6. $f(x)=1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+\ldots+\frac{(-1)^nx^{2n}}{(2n+1)!}+\ldots$

7. $f(x)=\frac{1}{x}+\frac{1}{2}+\frac{x}{4\cdot2!}+\frac{x^2}{8\cdot3!}+\ldots+\frac{x^{n-1}}{2^nn!}+\ldots$

8. 1

9. 1

10. 0.310

11. 0.160

12. -1

13. 4!

14. -9!

15. $f(x)=-\frac{x-3}{5}+\frac{(x-3)^2}{5^2\cdot2^2}-\frac{(x-3)^3}{5^3\cdot3^2}+\frac{(x-3)^4}{5^4\cdot4^2}-\ldots+\frac{(-1)^n(x-3)^{n+1}}{5^{n+1}\cdot2^{n+1}}+\ldots$

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