Question 2

A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t+1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12 \sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

(d) At what time t, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

(a) $\int_0^{12} H(t)\,dt = 70.570$ or 70.571	2 : { 1 : integral; 1 : answer }
(b) $H(6) - R(6) = -2.924$, so the level of heating oil is falling at $t = 6$.	1 : answer with reason
(c) $125 + \int_0^{12} (H(t) - R(t))\,dt = 122.025$ or 122.026	3 : { 1 : limits; 1 : integrand; 1 : answer }
(d) The absolute minimum occurs at a critical point or an endpoint. $H(t) - R(t) = 0$ when $t = 4.790$ and $t = 11.318$. The volume increases until $t = 4.790$, then decreases until $t = 11.318$, then increases, so the absolute minimum will be at $t = 0$ or at $t = 11.318$. $125 + \int_0^{11.318} (H(t) - R(t))\,dt = 120.738$ Since the volume is 125 at $t = 0$, the volume is least at $t = 11.318$.	3 : { 1 : sets $H(t) - R(t) = 0$; 1 : volume is least at $t = 11.318$; 1 : analysis for absolute minimum }

Question 2

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

(a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

(b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

(c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

(d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

Solution	Scoring
(a) $P'(9) = 1 - 3e^{-0.6} = -0.646 < 0$ so the amount is not increasing at this time.	1 : answer with reason
(b) $P'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0$ $t = (5\ln 3)^2 = 30.174$ $P'(t)$ is negative for $0 < t < (5\ln 3)^2$ and positive for $t > (5\ln 3)^2$. Therefore there is a minimum at $t = (5\ln 3)^2$.	3 { 1 : sets $P'(t) = 0$ 1 : solves for t 1 : justification
(c) $P(30.174) = 50 + \int_0^{30.174} \left(1 - 3e^{-0.2\sqrt{t}}\right) dt$ $= 35.104 < 40$, so the lake is safe.	3 { 1 : integrand 1 : limits 1 : conclusion with reason based on integral of $P'(t)$
(d) $P'(0) = 1 - 3 = -2$. The lake will become safe when the amount decreases by 10. A linear model predicts this will happen when $t = 5$.	2 { 1 : slope of tangent line 1 : answer

Question 2

For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

Solution	Points
(a) Since $R(6) = 4.438 > 0$, the number of mosquitoes is increasing at $t = 6$.	1 : shows that $R(6) > 0$
(b) $R'(6) = -1.913$ Since $R'(6) < 0$, the number of mosquitoes is increasing at a decreasing rate at $t = 6$.	2 : { 1 : considers $R'(6)$; 1 : answer with reason }
(c) $1000 + \int_0^{31} R(t)\,dt = 964.335$ To the nearest whole number, there are 964 mosquitoes.	2 : { 1 : integral; 1 : answer }
(d) $R(t) = 0$ when $t = 0$, $t = 2.5\pi$, or $t = 7.5\pi$ $R(t) > 0$ on $0 < t < 2.5\pi$ $R(t) < 0$ on $2.5\pi < t < 7.5\pi$ $R(t) > 0$ on $7.5\pi < t < 31$ The absolute maximum number of mosquitoes occurs at $t = 2.5\pi$ or at $t = 31$. $1000 + \int_0^{2.5\pi} R(t)\,dt = 1039.357$, There are 964 mosquitoes at $t = 31$, so the maximum number of mosquitoes is 1039, to the nearest whole number.	4 : { 2 : absolute maximum value (1 : integral; 1 : answer); 2 : analysis (1 : computes interior critical points; 1 : completes analysis) }

Question 1

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

(a) $\int_0^6 f(t)\,dt = 142.274$ or 142.275 cubic feet

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Rate of change is $f(8) - g(8) = -59.582$ or -59.583 cubic feet per hour.

$1 : \text{answer}$

(c) $h(0) = 0$

For $0 < t \le 6$, $h(t) = h(0) + \int_0^t g(s)\,ds = 0 + \int_0^t 0\,ds = 0.$

For $6 < t \le 7$, $h(t) = h(6) + \int_6^t g(s)\,ds = 0 + \int_6^t 125\,ds = 125(t-6).$

For $7 < t \le 9$, $h(t) = h(7) + \int_7^t g(s)\,ds = 125 + \int_7^t 108\,ds = 125 + 108(t-7).$

Thus, $h(t) = \begin{cases} 0 & \text{for } 0 \le t \le 6 \\ 125(t-6) & \text{for } 6 < t \le 7 \\ 125 + 108(t-7) & \text{for } 7 < t \le 9 \end{cases}$

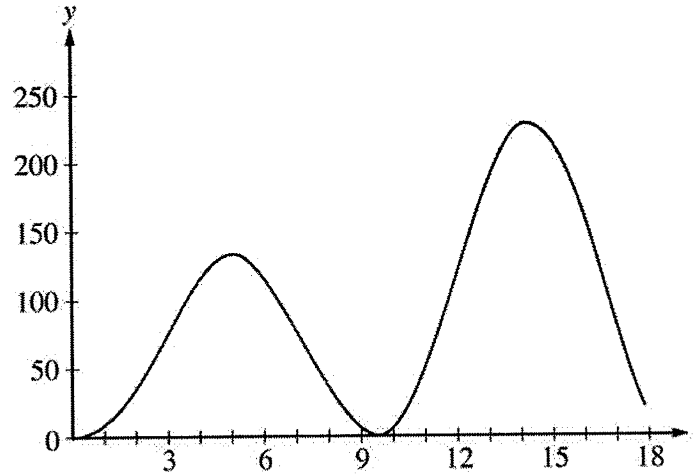
$3 : \begin{cases} 1 : h(t) \text{ for } 0 \le t \le 6 \\ 1 : h(t) \text{ for } 6 < t \le 7 \\ 1 : h(t) \text{ for } 7 < t \le 9 \end{cases}$

(d) Amount of snow is $\int_0^9 f(t)\,dt - h(9) = 26.334$ or 26.335 cubic feet.

$3 : \begin{cases} 1 : \text{integral} \\ 1 : h(9) \\ 1 : \text{answer} \end{cases}$

Question 2

At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.



(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

(a) $\int_0^{18} L(t)\,dt \approx 1658$ cars

$2 : \begin{cases} 1 : \text{setup} \\ 1 : \text{answer} \end{cases}$

(b) $L(t) = 150$ when $t = 12.42831,\ 16.12166$

Let $R = 12.42831$ and $S = 16.12166$

$L(t) \ge 150$ for t in the interval $[R, S]$

$\frac{1}{S - R}\int_R^S L(t)\,dt = 199.426$ cars per hour

$3 : \begin{cases} 1 : t\text{-interval when } L(t) \ge 150 \\ 1 : \text{average value integral} \\ 1 : \text{answer with units} \end{cases}$

(c) For the product to exceed 200,000, the number of cars turning left in a two-hour interval must be greater than 400.

$$\int_{13}^{15} L(t)\,dt = 431.931 > 400$$

$4 : \begin{cases} 1 : \text{considers 400 cars} \\ 1 : \text{valid interval } [h, h+2] \\ 1 : \text{value of } \int_h^{h+2} L(t)\,dt \\ 1 : \text{answer and explanation} \end{cases}$

OR

The number of cars turning left will be greater than 400 on a two-hour interval if $L(t) \ge 200$ on that interval.

$L(t) \ge 200$ on any two-hour subinterval of $[13.25304, 15.32386]$.

OR

$4 : \begin{cases} 1 : \text{considers 200 cars per hour} \\ 1 : \text{solves } L(t) \ge 200 \\ 1 : \text{discusses 2 hour interval} \\ 1 : \text{answer and explanation} \end{cases}$

Yes, a traffic signal is required.

Question 2

A water tank at Camp Newton holds 1200 gallons of water at time $t = 0$. During the time interval $0 \le t \le 18$ hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t}\sin^2\left(\frac{t}{6}\right) \text{ gallons per hour.}$$

During the same time interval, water is removed from the tank at the rate

$$R(t) = 275\sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

(a) Is the amount of water in the tank increasing at time $t = 15$? Why or why not?

(b) To the nearest whole number, how many gallons of water are in the tank at time $t = 18$?

(c) At what time t, for $0 \le t \le 18$, is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.

(d) For $t > 18$, no water is pumped into the tank, but water continues to be removed at the rate $R(t)$ until the tank becomes empty. Let k be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

(a) No; the amount of water is not increasing at $t = 15$ since $W(15) - R(15) = -121.09 < 0$.

1 : answer with reason

(b) $1200 + \int_0^{18} (W(t) - R(t))\,dt = 1309.788$

1310 gallons

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $W(t) - R(t) = 0$

$t = 0, 6.4948, 12.9748$

t (hours)	gallons of water
0	1200
6.495	525
12.975	1697
18	1310

The values at the endpoints and the critical points show that the absolute minimum occurs when $t = 6.494$ or 6.495.

$3 : \begin{cases} 1 : \text{interior critical points} \\ 1 : \text{amount of water is least at} \\ \quad t = 6.494 \text{ or } 6.495 \\ 1 : \text{analysis for absolute minimum} \end{cases}$

(d) $\int_{18}^{k} R(t)\,dt = 1310$

$2 : \begin{cases} 1 : \text{limits} \\ 1 : \text{equation} \end{cases}$

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Question 1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(a) $\int_0^{30} F(t)\,dt = 2474$ cars	$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(b) $F'(7) = -1.872$ or -1.873 Since $F'(7) < 0$, the traffic flow is decreasing at $t = 7$.	1 : answer with reason
(c) $\frac{1}{5}\int_{10}^{15} F(t)\,dt = 81.899$ cars/min	$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(d) $\dfrac{F(15) - F(10)}{15 - 10} = 1.517$ or 1.518 cars/min^2	1 : answer
Units of cars/min in (c) and cars/min^2 in (d)	1 : units in (c) and (d)