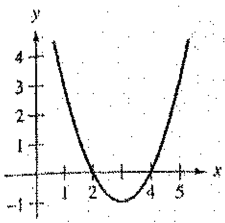
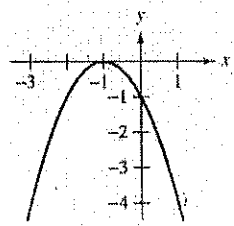
In Exercises 1–10, identify the open intervals on which the function is increasing or decreasing.

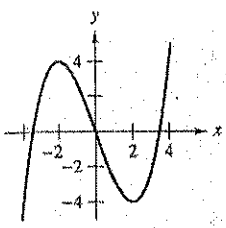
1. $f(x) = x^2 - 6x + 8$



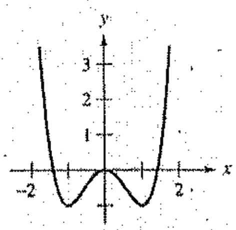
2. $y = -(x + 1)^2$



3. $y = \frac{x^3}{4} - 3x$



4. $f(x) = x^4 - 2x^2$



5. $f(x) = \frac{1}{x^2}$

6. $y = \frac{x^2}{x + 1}$

7. $g(x) = x^2 - 2x - 8$

8. $h(x) = 27x - x^3$

9. $y = x\sqrt{16 - x^2}$

10. $y = x + \frac{4}{x}$

In Exercises 11–32, find the critical numbers of f (if any). Find the open intervals on which the function is increasing or decreasing and locate all relative extrema. Use a graphing utility to confirm your results.

11. $f(x) = x^2 - 6x$

12. $f(x) = x^2 + 8x + 10$

13. $f(x) = -2x^2 + 4x + 3$

14. $f(x) = -(x^2 + 8x + 12)$

15. $f(x) = 2x^3 + 3x^2 - 12x$

16. $f(x) = x^3 - 6x^2 + 15$

17. $f(x) = x^2(3 - x)$

18. $f(x) = (x + 2)^2(x - 1)$

19. $f(x) = \frac{x^5 - 5x}{5}$

20. $f(x) = x^4 - 32x + 4$

21. $f(x) = x^{1/3} + 1$

22. $f(x) = x^{2/3} - 4$

23. $f(x) = (x - 1)^{2/3}$

24. $f(x) = (x - 1)^{1/3}$

25. $f(x) = 5 - |x - 5|$

26. $f(x) = |x + 3| - 1$

27. $f(x) = x + \frac{1}{x}$

28. $f(x) = \frac{x}{x + 1}$

29. $f(x) = \frac{x^2}{x^2 - 9}$

30. $f(x) = \frac{x + 3}{x^2}$

31. $f(x) = \frac{x^2 - 2x + 1}{x + 1}$

32. $f(x) = \frac{x^2 - 3x - 4}{x - 2}$

In Exercises 33–36, consider the function on the interval $(0, 2\pi)$. Find the open intervals on which the function is increasing or decreasing and locate all relative extrema. Use a graphing utility to confirm your results.

33. $f(x) = \frac{x}{2} + \cos x$

34. $f(x) = \sin x \cos x$

35. $f(x) = \sin^2 x + \sin x$

36. $f(x) = \frac{\sin x}{1 + \cos^2 x}$

In Exercises 37–40, (a) use a computer algebra system to differentiate the function, (b) sketch the graphs of f and f' on the same set of coordinate axes over the indicated interval, (c) find the critical numbers of f in the open interval, and (d) find the interval(s) on which f' is positive and the interval(s) on which it is negative. Compare the behavior of f and the sign of f'.

37. $f(x) = 2x\sqrt{9 - x^2}$, $[-3, 3]$

38. $f(x) = 10(5 - \sqrt{x^2 - 3x + 16})$, $[0, 5]$

39. $f(t) = t^2 \sin t$, $[0, 2\pi]$

40. $f(x) = \frac{x}{2} + \cos \frac{x}{2}$, $[0, 4\pi]$

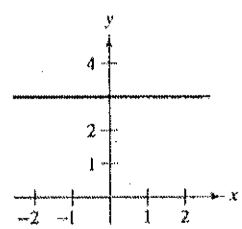
In Exercises 41 and 42, use symmetry, extrema, and zeros to sketch the graph of f. How do the functions f and g differ? Explain.

41. $f(x) = \frac{x^5 - 4x^3 + 3x}{x^2 - 1}$, $\quad g(x) = x(x^2 - 3)$

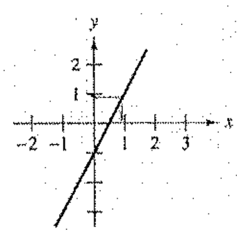
42. $f(t) = \cos^2 t - \sin^2 t$, $\quad g(t) = 1 - 2\sin^2 t$, $\quad (-2, 2)$

Think About It In Exercises 43–48, the graph of f is shown in the figure. Sketch a graph of the derivative of f. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

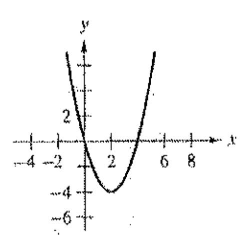
43.



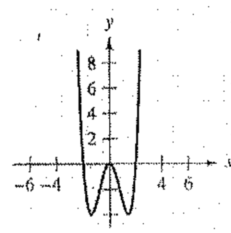
44.



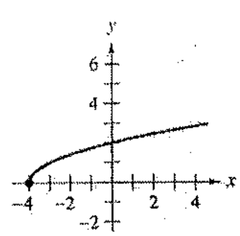
45.



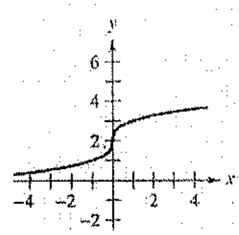
46.



47.



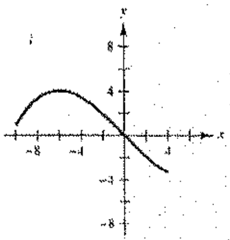
48.



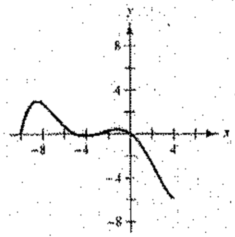
) f is continuous and changes signs in $[-10, 4]$ (Intermediate Value Theorem).

(b) There exist real numbers a and b such that $-10 < a < b < 4$ and $f(a) = f(b) = 2$. Therefore, f' has a zero in the interval by Rolle's Theorem.

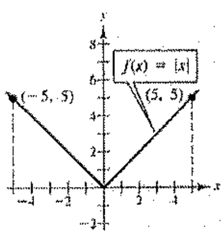
(c)



(d)



(e) No, by Theorem 2.1.



. False. f is not continuous on $[-1, 1]$.

. True **57.** Proof **59.** Proof **61.** Proof

ection 3.3 (page 181)

ncreasing on $(3, \infty)$; Decreasing on $(-\infty, 3)$

. Increasing on $(-\infty, -2)$ and $(2, \infty)$; Decreasing on $(-2, 2)$

. Increasing on $(-\infty, 0)$; Decreasing on $(0, \infty)$

. Increasing on $(1, \infty)$; Decreasing on $(-\infty, 1)$

. Increasing on $(-2\sqrt{2}, 2\sqrt{2})$
Decreasing on $(-4, -2\sqrt{2}), (2\sqrt{2}, 4)$

. Critical number: $x = 3$
Increasing on $(3, \infty)$
Decreasing on $(-\infty, 3)$
Relative minimum: $(3, -9)$

. Critical number: $x = 1$
Increasing on $(-\infty, 1)$
Decreasing on $(1, \infty)$
Relative maximum: $(1, 5)$

. Critical numbers: $x = -2, 1$
Increasing on $(-\infty, -2)$ and $(1, \infty)$
Decreasing on $(-2, 1)$
Relative maximum: $(-2, 20)$
Relative minimum: $(1, -7)$

. Critical numbers: $x = 0, 2$
Increasing on $(0, 2)$
ecreasing on $(-\infty, 0), (2, \infty)$
elative maximum: $(2, 4)$
Relative minimum: $(0, 0)$

19. Critical numbers: $x = -1, 1$
Increasing on $(-\infty, -1)$ and $(1, \infty)$
Decreasing on $(-1, 1)$
Relative maximum: $\left(-1, \frac{4}{5}\right)$
Relative minimum: $\left(1, -\frac{4}{5}\right)$

21. Critical number: $x = 0$
Increasing on $(-\infty, \infty)$
No relative extrema

23. Critical number: $x = 1$
Increasing on $(1, \infty)$
Decreasing on $(-\infty, 1)$
Relative minimum: $(1, 0)$

25. Critical number: $x = 5$
Increasing on $(-\infty, 5)$
Increasing on $(-\infty, 5)$
Relative maximum: $(5, 5)$

27. Critical numbers: $x = -1, 1$
Discontinuity: $x = 0$
Increasing on $(-\infty, -1)$ and $(1, \infty)$
Decreasing on $(-1, 0)$ and $(0, 1)$
Relative maximum: $(-1, -2)$
Relative minimum: $(1, 2)$

29. Critical number: $x = 0$
Discontinuities: $x = -3, 3$
Increasing on $(-\infty, -3)$ and $(-3, 0)$
Decreasing on $(0, 3)$ and $(3, \infty)$
Relative maximum: $(0, 0)$

31. Critical numbers: $x = -3, 1$
Discontinuity: $x = -1$
Increasing on $(-\infty, -3)$ and $(1, \infty)$
Decreasing on $(-3, -1)$ and $(-1, 1)$
Relative maximum: $(-3, -8)$
Relative minimum: $(1, 0)$

33. Critical numbers: $x = \frac{\pi}{6}, \frac{5\pi}{6}$
Increasing on $\left(0, \frac{\pi}{6}\right), \left(\frac{5\pi}{6}, 2\pi\right)$
Decreasing on $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$
Relative maximum: $\left(\frac{\pi}{6}, \frac{[\pi + 6\sqrt{3}]}{12}\right)$
Relative minimum: $\left(\frac{5\pi}{6}, \frac{[5\pi - 6\sqrt{3}]}{12}\right)$

35. Critical numbers: $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$
Increasing on $\left(0, \frac{\pi}{2}\right), \left(\frac{7\pi}{6}, \frac{3\pi}{2}\right), \left(\frac{11\pi}{6}, 2\pi\right)$
Decreasing on $\left(\frac{\pi}{2}, \frac{7\pi}{6}\right), \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$
Relative maxima: $\left(\frac{\pi}{2}, 2\right), \left(\frac{3\pi}{2}, 0\right)$
Relative minima: $\left(\frac{7\pi}{6}, -\frac{1}{4}\right), \left(\frac{11\pi}{6}, -\frac{1}{4}\right)$