Worksheet 1. What You Need to Know About Motion Along the x-axis (Part 1)

In discussing motion, there are three closely related concepts that you need to keep straight. These are:

- **Position**: \( x(t) \) — determines where the particle is located on the x-axis at a given time \( t \).
- **Velocity**: \( v(t) = x'(t) \) — determines how fast the position is changing at a time \( t \) as well as the direction of movement.
- **Acceleration**: \( a(t) = v'(t) = x''(t) \) — how fast the velocity is changing at time \( t \); the sign indicates if the velocity is increasing or decreasing.

If \( x(t) \) represents the position of a particle along the x-axis at any time \( t \), then the following statements are true.

1. "Initially" means when \( t = 0 \).
2. "At the origin" means \( x(t) = 0 \).
3. "At rest" means \( v(t) = 0 \).
4. If the velocity of the particle is positive, then the particle is moving to the right.
5. If the velocity of the particle is negative, then the particle is moving to the left.
6. To find average velocity over a time interval, divide the change in position by the change in time.
7. Instantaneous velocity is the velocity at a single moment (instantly) in time.
8. If the acceleration of the particle is positive, then the velocity is increasing.
9. If the acceleration of the particle is negative, then the velocity is decreasing.
10. In order for a particle to change direction, the velocity must change signs.
11. One way to determine total distance traveled over a time interval is to find the sum of the absolute values of the differences in position between all resting points.

Here's an example: If the position of a particle is given by:

\[
x(t) = \frac{1}{3}t^3 - t^2 - 3t + 4,
\]

find the total distance traveled on the interval \( 0 \leq t \leq 6 \).
Worksheet 2. Sample Practice Problems for the Topic of Motion (Part 1)

Example 1 (numerical).
The data in the table below give selected values for the velocity, in meters/minute, of a particle moving along the x-axis. The velocity $v$ is a differentiable function of time $t$.

<table>
<thead>
<tr>
<th>Time $t$ (min)</th>
<th>0</th>
<th>2</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity $v(t)$ (meters/min)</td>
<td>-3</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

1. At $t = 0$, is the particle moving to the right or to the left? Explain your answer.

   left, $v(t)$ is negative

2. Is there a time during the time interval $0 \leq t \leq 12$ minutes when the particle is at rest? Explain your answer.

   yes, since the velocity is differentiable it is also continuous. By the IVT, since the velocity goes from $0$ to $0$, it must cross zero which means there is one point when the particle is at rest.

3. Use data from the table to find an approximation for $v'(10)$ and explain the meaning of $v'(10)$ in terms of the motion of the particle. Show the computations that lead to your answer and indicate units of measure.

   $v'(10) = \frac{v(12) - v(8)}{12 - 8} = \frac{-1}{2} \text{ m/min}^2$

   $v'(10)$ is the acceleration of the particle at $t = 10 \text{ min}$

4. Let $a(t)$ denote the acceleration of the particle at time $t$. Is there guaranteed to be a time $t = c$ in the interval $0 \leq t \leq 12$ such that $a(c) = 0$? Justify your answer.

   yes, because $v(t)$ is diff & cont & $f(b) = f(c)$, by Rolle's theorem there must be a value $c$ in $(a,c)$ s.t. $v'(c) = 0$
Example 2 (graphical).
The graph below represents the velocity $v$, in feet per second, of a particle moving along the $x$-axis over the time interval from $t = 0$ to $t = 9$ seconds.

1. At $t = 4$ seconds, is the particle moving to the right or left? Explain your answer.
   \[ \text{right bc } v(t) \text{ is } >0 \]

2. Over what time interval is the particle moving to the left? Explain your answer.
   \[ (5, 9) \text{ be } v(t) \text{ is } <0 \]

3. At $t = 4$ seconds, is the acceleration of the particle positive or negative? Explain your answer.
   \[ \text{negative bc the slope at } t=4 \text{ of } v(t) \text{ is negative} \]

4. What is the average acceleration of the particle over the interval $2 \leq t \leq 4$? Show the computations that lead to your answer and indicate units of measure.
   \[ \frac{v(4) - v(2)}{4-2} = \frac{6 - 9}{4-2} = -\frac{3}{2} \text{ ft/sec}^2 \]

5. Is there guaranteed to be a time $t$ in the interval $2 \leq t \leq 4$ such that $v'(t) = -3/2$ ft/sec$^2$? Justify your answer.
   \[ \text{no, it is not guaranteed bc } v(t) \text{ does not apply here bc the fn is not diff @ } t=3. \]
6. At what time $t$ in the given interval is the particle farthest to the right? Explain your answer.

when $t=5$ bc after $t=5$ the particle changed direction & goes left.

Example 3 (analytic).
A particle moves along the $x$-axis so that at time $t$ its position is given by:

$$x(t) = t^3 - 6t^2 + 9t + 11$$
$$v(t) = 3t^2 - 12t + 9$$
$$a(t) = 6t - 12$$

1. At $t = 0$, is the particle moving to the right or to the left? Explain your answer.

$v(0) = 9$, @ $t=0$, the particle is moving to the right bc $v(0)$ is positive.

2. At $t = 1$, is the velocity of the particle increasing or decreasing? Explain your answer.

$a(1) = -6$, @ $t=1$, the velocity of the particle is decreasing bc $a(1)$ is negative.

3. Find all values of $t$ for which the particle is moving to the left.

$$0 = 3(t^2 - 4t + 3)$$
$$0 = 3(t-3)(t-1)$$
$$t = 3, t = 1$$

(1.3) bc $v(t)$ is ⊗

4. Find the total distance traveled by the particle over the time interval $0 \leq t \leq 5$.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$x(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>31</td>
</tr>
</tbody>
</table>

28 units
Worksheet 3. Understanding the Relationships Among Velocity, Speed, and Acceleration

Speed is the absolute value of velocity. It tells you how fast something is moving without regard to the direction of movement.

1. What effect does absolute value have on numbers?
   
   makes them positive or zero.

2. What effect does taking the absolute value of a function have on its graph?
   
   reflects all values that are below the x-axis, above the x-axis.

For each situation below, the graph of a differentiable function giving velocity as a function of time \( t \) is shown for \( 1 \leq t \leq 5 \), along with selected values of the velocity function. In the graph, each horizontal grid mark represents 1 unit of time and each vertical grid mark represents 4 units of velocity. For each situation, plot the speed graph on the same coordinate plane as the velocity graph and fill in the speed values in the table. Then, answer the questions below based on both the graph and the table of values.

**Situation 1: Velocity graph**

![Velocity graph]

<table>
<thead>
<tr>
<th>time</th>
<th>velocity</th>
<th>speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td>16</td>
</tr>
</tbody>
</table>

In this situation, the velocity is \( + \) and \( \text{incr} \).

Positive or negative? Increasing or decreasing?

Because velocity is \( \text{incr.} \), we know acceleration is \( + \).

Increasing or decreasing? Positive or negative?

By examining the graph of speed and the table of values, we can conclude that speed is \( \text{incr.} \).

Increasing or decreasing?
Situation 2: Velocity graph

<table>
<thead>
<tr>
<th>time</th>
<th>velocity</th>
<th>speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>-4</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>-8</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>-16</td>
<td>16</td>
</tr>
</tbody>
</table>

In this situation, the velocity is \( \Theta \) and \( \text{decr.} \).
Positive or negative? Increasing or decreasing?

Because velocity is \( \text{decr.} \), we know acceleration is \( \Theta \).
Increasing or decreasing? Positive or negative?

By examining the graph of speed and the table of values, we can conclude that speed is \( \text{increasing} \).
Increasing or decreasing?

Situation 3: Velocity graph

<table>
<thead>
<tr>
<th>time</th>
<th>velocity</th>
<th>speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-16</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>-8</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>-4</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>

In this situation, the velocity is \( \Theta \) and \( \text{incr} \).
Positive or negative? Increasing or decreasing?

Because velocity is \( \text{incr} \), we know acceleration is \( \Theta \).
Increasing or decreasing? Positive or negative?

By examining the graph of speed and the table of values, we can conclude that speed is \( \text{decr.} \).
Increasing or decreasing?
Situation 4: Velocity graph

<table>
<thead>
<tr>
<th>time</th>
<th>velocity</th>
<th>speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

In this situation, the velocity is _decr._ and _decr._

Positive or negative? Increasing or decreasing?

Because velocity is _decr._, we know acceleration is _decr._

Increasing or decreasing? Positive or negative?

By examining the graph of speed and the table of values, we can conclude that speed is _decr._

Increasing or decreasing?

**Conclusion:**

In which situations was the speed increasing? when \( |v| > 0 \) (#51, 2)

When the speed is increasing, the velocity and acceleration have same signs.

Same or opposite?

In which situations was the speed decreasing? when \( v \) and \( a \) have opp signs

When the speed is decreasing, the velocity and acceleration have opp signs.

Same or opposite?
Assessing Students’ Understanding (A Short Quiz):

1. If velocity is negative and acceleration is positive, then speed is **decreasing**.

2. If velocity is positive and speed is decreasing, then acceleration is **negative**.

3. If velocity is positive and decreasing, then speed is **decreasing**.

4. If speed is increasing and acceleration is negative, then velocity is **negative**.

5. If velocity is negative and increasing, then speed is **decreasing**.

6. If the particle is moving to the left and speed is decreasing, then acceleration is **positive**. velocity = 0

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