How to justify in AP Calculus AB

Stuff to remember:
Critical values/numbers are where \( f'(x) \) is either equal to zero OR \( f'(x) \) is undefined [but \( f(x) \) is defined]
Possible points of inflection are where \( f''(x) \) is either equal to zero OR \( f''(x) \) is undefined [but \( f(x) \) is defined]
If a function is differentiable at a point, then the function is continuous at that point.

How to justify: PLEASE LOOK AT THE INTERVAL NOTATION
[Assume that the function is continuous and differentiable]
If \( f'(x) > 0 \) for every \( x \) in \((a, b)\), then \( f(x) \) is increasing on \([a, b]\)
If \( f'(x) < 0 \) for every \( x \) in \((a, b)\), then \( f(x) \) is increasing on \([a, b]\)
If \( f''(x) > 0 \) for every \( x \) in \((a, b)\), then \( f(x) \) is concave up on \((a, b)\) AND \( f'(x) \) is increasing on \([a, b]\)
If \( f''(x) < 0 \) for every \( x \) in \((a, b)\), then \( f(x) \) is concave down on \((a, b)\) AND \( f'(x) \) is decreasing on \([a, b]\)
If \( f'(x) \) is increasing on \((a, b)\), then \( f(x) \) is concave up on \((a, b)\) AND \( f''(x) > 0 \) on \((a, b)\)
If \( f'(x) \) is decreasing on \((a, b)\), then \( f(x) \) is concave down on \((a, b)\) AND \( f''(x) < 0 \) on \((a, b)\)
Let \( x = a \) be a critical value for \( f(x) \) [in other words, \( f'(a) = 0 \) OR \( f'(a) \) is undefined but \( f(a) \) is defined]

To show that \( (a, f(a)) \) is a relative/local minimum:
"At \( x = a \), \( f'(a) \) changes from negative to positive values. Hence, \( f \) has a relative minimum at \( x = a \)."

To show that \( (a, f(a)) \) is a relative/local maximum:
"At \( x = a \), \( f'(a) \) changes from positive to negative values. Hence, \( f \) has a relative maximum at \( x = a \)."

Let \( f''(b) = 0 \) or \( f''(b) \) be undefined but \( f(b) \) is defined
To show that \( (b, f(b)) \) is a point of inflection you must justify with one of the following statements:
"At \( x = b \), \( f''(b) \) changes from positive to negative values. Hence, \( f \) has a point of inflection at \( x = b \)."

OR
"At \( x = b \), \( f''(b) \) changes from negative to positive values. Hence, \( f \) has a point of inflection at \( x = b \)."

If given the graph of the first derivative, then a point of inflection will occur at \( x = a \) if at \( x = a \) the graph of \( f'(x) \) changes from either increasing to decreasing OR decreasing to increasing.

NOTE: When you justify, you must be SPECIFIC. Do NOT state a rule but rather, explain what is occurring at some specific value of \( x = a \).
To justify an absolute maximum or absolute minimum
The candidates are the critical values of \( f(x) \) AND the endpoints!
First justify any relative min/max AND find their function values.
Then compare the function values to the endpoint values and
decide which is the absolute min/max.

Extreme Value Theorem
If a function, \( f \), is continuous on \([a, b]\), then \( f \) has both a maximum
and a minimum value on \([a, b]\).

Mean Value Theorem
Your justification statement should look like:
By the Mean Value Theorem there is a \( c \), \( a < c < b \), such that
\[
\frac{f(b) - f(a)}{b - a} = f'(c)
\]
You need to supply all of the values!

You never really need Rolle’s Theorem since you can always use the
Mean Value Theorem
Rolle’s Theorem [A “special” case of MVT]
Your justification statement should look like:
By Rolle’s Theorem, since \( f \) is continuous on \([a, b]\) and differentiable on \((a, b)\) AND \( f(a) = f(b) \), then there is a \( c \), \( a < c < b \), such that \( f'(c) = 0 \).
Note: You can always just use MVT.

Horizontal TANGENTS occur at \( x = a \) if \( f'(a) = 0 \)
Vertical TANGENTS occur at \( x = a \) if \( f'(a) \) is undefined but \( f(a) \) is defined

If \( f'(x) \) is of the form \( f'(x) = \frac{g(x)}{h(x)} \), then \( f \) will have a horizontal tangent when \( g(x) = 0 \) AND \( f \) will have a vertical tangent if \( h(x) = 0 \). [Of course \( f(x) \) must be defined for those values]
Horizontal Asymptote

\( y = b \) is a horizontal asymptote of the graph of \( y = f(x) \) if either

\[
\lim_{x \to \infty} f(x) = b \quad \text{OR} \quad \lim_{x \to -\infty} f(x) = b
\]

Limits of Rational Functions as \( x \to \pm \infty \) [End Behavior]

\[
\lim_{x \to \pm \infty} \frac{f(x)}{g(x)} = 0 \quad \text{if the degree of } f(x) < \text{the degree of } g(x)
\]

\[
\lim_{x \to \pm \infty} \frac{f(x)}{g(x)} = \infty \quad \text{or "dne" if the degree of } f(x) > \text{the degree of } g(x)
\]

\[
\lim_{x \to \pm \infty} \frac{f(x)}{g(x)} \quad \text{is finite} \quad \text{[there is a horizontal asymptote] if degree of } f(x) = \text{degree of } g(x)
\]

Remember: A graph is NOT justification. You must use Calculus to justify any extrema, points of inflection, horizontal asymptotes, etc.
Remember:
If $s(t)$ is the position function, then
Velocity, $v(t) = s'(t)$
Acceleration, $a(t) = v'(t) = s''(t)$
Speed is $|v(t)|$
The speed of a particle is increasing at $x = c$ if $v(c)$ and $a(c)$ are either both positive or both negative. The speed of a particle is decreasing at $x = c$ if $v(c)$ and $a(c)$ have different signs.
To justify that the speed of a particle is increasing at $x = c$ you need to clearly state that $v(c) > 0$ AND $a(c) > 0$ OR $v(c) < 0$ AND $a(c) < 0$ in a statement.
To justify that the speed of a particle is decreasing at $x = c$ you need to clearly state that $v(c) > 0$ AND $a(c) < 0$ OR $v(c) < 0$ AND $a(c) > 0$ in a statement.

OPTIMIZATION
If necessary, you might want to draw a picture of what the problem is about.
Figure out what you are trying to optimize.
Find a primary equation that fits what you are trying to optimize.
If necessary, find a secondary equation that will allow you to rewrite your primary equation in terms of just ONE VARIABLE.
Now just find the min or max using standard Calculus min/max techniques.