Chapter 6 Mixed Review

1. A woman is walking along a straight path beginning at point $P$. A graph of the walker’s velocity is given below in miles per hour. How far away from the starting point $P$ has she walked 9 hours from the starting time?
   
   a. 6.5 miles  
   b. 12 miles  
   c. 13.5 miles  
   d. 18 miles  
   e. 19.5 miles

2. \[
\int \frac{15x^9 + 4x^2 + 16}{x^5} \, dx
\]

3. Find the average value. Let $y = 2 \cos x$ on \([0, \frac{\pi}{2}]\)

4. Find the average rate of change for the equation in #3 on the same interval.
1091. The graph of a function $f$ consists of a semicircle and two line segments as shown below. Let $g(x) = \int_{1}^{x} f(t) \, dt$.

a) Find $g(1)$.

b) Find $g(3)$.

c) Find $g(-1)$.

d) Find all the values of $x$ on the open interval $(-3, 4)$ at which $g$ has a relative maximum.

e) Write an equation for the line tangent to the graph of $g$ at $x = -1$.

f) Find the $x$-coordinate of each point of inflection of the graph of $g$ on the open interval $(-3, 4)$.

g) Find the range of $g$.

6.

A car’s velocity is shown on the graph above. Which of the following gives the total distance traveled from $t = 0$ to $t = 16$ (in kilometers)?

(A) 360  (B) 390  (C) 780  (D) 1000  (E) 1360
7. A particle moves along the x-axis so that its acceleration at any time \( t > 0 \) is given by \( a(t) = 12t - 18 \). At time \( t = 1 \), the velocity of the particle is \( v(1) = 0 \) and the position is \( x(1) = 9 \).

(a) Write an expression for the velocity of the particle \( v(t) \).

(b) At what values of \( t \) does the particle change direction?

(c) Write an expression for the position \( x(t) \) of the particle.

(d) Find the total distance traveled by the particle from \( t = \frac{3}{2} \) to \( t = 6 \).

8. A continuous function \( f \) is defined on the closed interval \(-10 \leq x \leq 10\). The graph of \( f \) consists of a semi-circle and four line segments as shown in the figure above. Let \( g \) be the function defined by \( g(x) = \int_{-3}^{x} f(t) \, dt \).

(a) Find \( \lim_{x \to 5^-} g(x) \)

(b) Find the average rate of change for \( f \) on the interval \(-10 \leq x \leq 10\)

(c) Does the Mean Value Theorem guarantee a value \( c, -10 < c < 10 \) such that \( f'(c) \) will equal the average rate of change from part (b)?

(d) Show [using Calculus] that \( f'(6) \) does not exist

(e) Find the value of \( g(-3) \)

(f) Find the value of \( g(3) \)

(g) Find the value of \( g(-10) \)

(h) Find the value of \( g(10) \)
(i) Find $g'(x)$

(j) Find the $x-$value(s) of the critical value(s) for the graph of $g$ and classify as relative minimum, relative maximum, or neither

(k) Find all intervals where the graph of $g$ is increasing

(l) Find all intervals where the graph of $g$ is decreasing

(m) Find the absolute extrema for the graph of $g$

(n) Find $g'(5)$

(o) Write the equation of the line tangent to the graph of $g$ at $x = 5$

(p) Use the tangent line from part (o) to estimate $g(5.1)$

(q) Does the tangent line from part (o) lie above or below the graph of $g$?

(r) Is the estimate you found in part (p) an over- or an under-estimate?

(s) Find $g'(-4)$

(t) Write the equation of the line tangent to the graph of $g$ at $x = -4$

(u) Use the tangent line from part (t) to find an estimate for $g(-4.1)$

(v) Does the tangent line from part (t) lie above or below the graph of $g$?
13. If \( h'(x) = e^{x-1}(2x-1)^2(x-3)^3(4x+5) \), then \( h(x) \) has how many points of inflection?
   (A) 4
   (B) 3
   (C) 2
   (D) 1
   (E) 0

22. Let \( y = 2x(\sin 2x + x \cos 2x) \) in the interval \( 0 \leq x \leq \frac{\pi}{2} \). What is the average rate of change of \( y \) with respect to \( x \) in this interval?
   (A) \(-\pi\)
   (B) \(-\frac{\pi}{2}\)
   (C) 0
   (D) \(\frac{\pi}{2}\)
   (E) \(\pi\)

24. The concentration of an anti-inflammatory drug in the bloodstream \( t \) mins after taking a single dose is \( C(t) = \frac{2t}{8100 + t^2} \), \( t \geq 0 \).
   At what time is the concentration the greatest?
   (A) 90 minutes
   (B) 30\(\sqrt{6}\) minutes
   (C) 30\(\sqrt{3}\) minutes
   (D) 15\(\sqrt{6}\) minutes
   (E) none of these

29. Suppose \( g(0) = 4 \), \( g'(0) = 8 \), and \( g''(0) = -12 \). If \( h(x) = \sqrt{g(x)} \), what is \( h''(0) \)?
   (A) \(-5\)
   (B) \(-\frac{13}{4}\)
   (C) \(-\frac{1}{32}\)
   (D) \(\frac{3}{8}\)
   (E) 1

The graph of \( f(x) \) consists of four line segments and a semicircle as shown above in the closed interval \(-3 \leq x \leq 5\). Let \( g \) be the function given by \( g(x) = \int_{0}^{x} f(t) \, dt \). Use this information for problems 5–7.

5. What is \( g(-1) + g'(-1) + g''(-1) \)?
   (A) \(-1\)
   (B) 0
   (C) 1
   (D) 2
   (E) 3

6. What is \( \int_{-3}^{3} f(t) \, dt \)?
   (A) \(7 - \pi\)
   (B) \(7 - \frac{\pi}{2}\)
   (C) \(7 - \frac{\pi}{4}\)
   (D) \(12 - \frac{\pi}{2}\)
   (E) \(12 - \frac{\pi}{4}\)

7. Which of the following statements is false for \( g(x) \)?
   (A) The absolute maximum for \( g(x) \) occurs at \( x = 5 \).
   (B) A relative minimum for \( g(x) \) occurs at \( x = -1 \).
   (C) A point of inflection for \( g(x) \) occurs at \( x = 3 \).
   (D) \( g(x) \) has roots at \( x = 0 \) and \( x = -2 \).
   (E) \( g(x) \) is concave down in the open interval \(-2 < x < -1 \).
Let \( f(x) = x^2 + \int_{-2}^{x} g(t) \, dt \), where \( g(x) \) is shown in the graph above. Use this graph to answer problems 39–41.

39. What is \( f(-2) \)?

(A) -6
(B) -4
(C) 0
(D) 2
(E) 4

40. What is \( f'(-2) \)?

(A) -6
(B) -4
(C) 0
(D) 2
(E) 4

41. What is \( f'(2) \)?

(A) -6
(B) -4
(C) 0
(D) 2
(E) 4

5. Consider the function \( h(x) = 3x^2 - \sqrt{x+1} \).

a. Evaluate \( \frac{1}{3-\frac{-1}{2}} \int_{-1}^{3} (3x^2 - \sqrt{x+1}) \, dx \) and interpret its meaning.

b. What is the equation of the tangent to \( h(x) \) at \( x = 0 \)?

c. Use the tangent found in part b to approximate \( h(x) \) at \( x = -0.01 \).

d. Is the approximation, found in part c, greater or less than the actual value of \( h(x) \) at \( x = -0.01 \)? Justify your answer using calculus.