Calculus AB

Chapter 5 Part 1 Test Review (Attn: This review DOES NOT include Section 5.6)

1. When looking for absolute extrema, where do the possible extrema exist, and how do you find them?
   Absolute extreme exist at end points & critical points. To find them: 1) Find critical points where f'=0 & f' is undefined. 2) List and test these candidates by plugging the x-values into the original function. 3) Determine abs max/abs min.

2. How do you justify relative extrema?
   1st Derivative Test: Look for values where f' changes sign. If f' changes from neg to pos, there is a rel min. If f' changes from pos to neg, there is a rel max.
   2nd Derivative Test: Find where f'' is 0 or und, determine sign of f'' at these x-values. If f'' is pos - rel max, if f'' is neg - rel min.

3. How do you justify that a function is increasing or decreasing?
   f increases when f' > 0; f decreases when f' < 0.

4. How do you justify that a function is concave up or concave down?
   f is concave up when f'' > 0; f is concave down when f'' < 0.

5. How do you justify that a function has a point of inflection?
   f has a point of inflection at x = c when f'' changes signs at x = c.

6. Using the graph of g(x) below, determine the signs of g'(x) and g''(x) at each point. Explain your reasoning.

   At x = a … g(x) is decreasing so g'(x) < 0 and g(x) is concave up so g''(x) > 0.
   At x = b … g(x) has a horizontal tangent line so g'(x) = 0 and g(x) is concave up so g''(x) > 0.
   At x = c … g'(x) = 0 because g(x) has a horizontal tangent line and g''(x) < 0 because g(x) is concave down.
   At x = d … g'(x) < 0 because g(x) is decreasing and g'' < 0 because g(x) is concave down.

7. Given the graph of f' below answer each of the following questions, and justify your response with a statement that contains the phrase “since f’________________________…”

   a) When is f increasing? 
      On (a, b) U (d, e) bc f' > 0.

   b) When is f decreasing? 
      On (b, d) bc f' < 0.

   c) When is f concave up? 
      On (c, e) bc f'' is positive (or bc f' is increasing)

   d) When is f concave down? 
      On (a, c) bc f'' is negative (or bc f' is decreasing)

   e) When does f have a relative maximum? 
      Rel max at x = b bc f' changes from pos to neg.

   f) When does f have a relative minimum? 
      Rel min at x = d bc f' changes from neg to pos.

   g) When does f have a point of inflection? 
      f has a point of inflection at x = c bc f'' changes sign. (or bc f' changes from decreasing to increasing).
8. Find the value of \( c \) guaranteed by the MVT for \( f(x) = 4x^2 + 5x \) on the interval \([-2, 1]\).

9. [Calculator Allowed] Find the value of \( c \) guaranteed by the MVT for \( f(x) = \sin x \) on the interval \([4, 5]\).

   \[ f'(c) = \frac{f(5) - f(4)}{5-4} \]
   \[ f'(x) = \cos x \]
   \[ c = \cos^{-1} (\sin 5 - \sin 4) \]
   \[ = 1.774 \quad \text{not in interval } [4, 5] \]

   \[ 2\pi - 1.774 = 4.509 \]

10. Suppose \( y = x^3 - 3x \). [No Calculator]

   a) Find the zeros of the function.
   \[ x = 0, x = \pm \sqrt{3} \]

   b) Determine where \( y \) is increasing or decreasing and justify your response.
   \[ y' = 3x^2 - 3 \]
   \[ y' = 0 \Rightarrow x = \pm 1 \quad \text{critical points} \]
   \[ y \text{ is increasing on } (-\infty, -1) \cup (1, \infty) \] (since \( f' > 0 \).)
   \[ y \text{ is decreasing on } (-1, 1) \] (since \( f' < 0 \).)

   c) Determine all local extrema and justify your response.
   \[ f(-1) = 2 \]
   \[ f(1) = -2 \]
   \[ y \text{ has a relative max of 2 at } x = -1 \] (since \( f' \) changes from + to -).
   \[ y \text{ has a relative min of -2 at } x = 1 \] (since \( f' \) changes from - to +).

   d) Determine the points where \( y \) is concave up or concave down, and find any points of inflection. Justify your responses.
   \[ y'' = 6x \]
   \[ y'' = 0 \Rightarrow x = 0 \]
   \[ f(0) = 0 \]
   \[ y \text{ is concave down on } (-\infty, 0) \] (since \( y'' < 0 \).)
   \[ y \text{ is concave up on } (0, \infty) \] (since \( y'' > 0 \).)
   \[ y \text{ has a point of inflection at } (0, 0) \] (since \( y'' \) changes sign.)

   e) Use all your information to sketch a graph of this function.

12. If \( f'(x) = x^2 - 9x + 1 \), what does \( f(x) \) equal?

   \[ f(x) = \frac{1}{3} x^3 - \frac{9}{2} x^2 + x + C \]
13. Suppose \( \frac{d^2y}{dx^2} = x^3 - 4x^2 \). Justify each response below.

a) Where is \( y \) concave up?

\[ x^2(x-4) = 0 \]
\[ x = 0, x = 4 \]

\( y \) is concave up on \((4,\infty)\) bc \( y'' > 0 \).

b) Where is \( y \) concave down?

\( y \) is concave down on \((-\infty,0) \cup (0,4)\) bc \( y'' < 0 \).

c) Are there any inflection points on \( y \)? If so, where?

\( y \) has a point on inflection when \( x = 4 \) bc \( y'' \) changes sign.

14. [Calculator Allowed] The derivative of \( h(x) \) is given by \( h'(x) = 2\cos(x - \frac{x}{2}) + 1 \) on the interval \([-2\pi, 2\pi]\).

Justify EVERY response.

a) Where is \( h(x) \) increasing?

\( h(x) \) is incr. on \((2\pi, -3.665) \cup (-1.571, 2.618) \cup (4.712, 2\pi)\) bc \( h'(x) > 0 \).

b) Where is \( h(x) \) concave down?

\( h(x) \) is concave down on \((-5.760, -2.618) \cup (-.524, 3.665)\) bc \( h''(x) < 0 \).

c) Find the \( x \)-coordinates of all extrema of \( h(x) \) on the interval \([-2\pi, 2\pi]\).

\( h'(x) = 0 \)
when \( x = -3.445, -1.571, 2.618, 4.712 \)
endpoints \( x = -2\pi, 2\pi \)

rel min \( @ \ x = -2\pi, -1.571, 4.712 \) bc \( h(x) \) change from \( @+ \to @ \)

rel max \( @ \ x = -3.445, 2.618, 2\pi \) bc \( h(x) \) change from \( @- \to @ \).

d) Does \( h(x) \) have a point(s) of inflection? If so, where?

@ \( x = -5.760, .524, -2.618, 3.665 \) bc \( h''(x) \) changes sign.

15. A rectangle is inscribed between the parabolas \( y = 4x^2 \) and \( y = 30 - x^2 \) as shown in the picture.

What is the maximum area of such a rectangle? Justify your response using CALCULUS.
16. **USE CALCULUS:** Find the maximum area of a rectangle inscribed under the curve \( f(x) = \sqrt{16-x^2} \).

17. [Calculator Allowed] **USE CALCULUS:** A rectangle is inscribed under one arch of \( y = 8\cos(0.3x) \) with its base on the \( x \)-axis and its upper two vertices on the curve symmetric about the \( y \)-axis. What is the largest area the rectangle can have?

18. The function \( f \) is continuous on \([0,3]\) and satisfies the following:

<table>
<thead>
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<th>( x )</th>
<th>0</th>
<th>( 0 &lt; x &lt; 1 )</th>
<th>1</th>
<th>( 1 &lt; x &lt; 2 )</th>
<th>2</th>
<th>( 2 &lt; x &lt; 3 )</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f' )</td>
<td>0</td>
<td>Neg</td>
<td>(-2)</td>
<td>Neg</td>
<td>0</td>
<td>Pos</td>
<td>3</td>
</tr>
<tr>
<td>( f'' )</td>
<td>(-3)</td>
<td>Neg</td>
<td>0</td>
<td>Pos</td>
<td>DNE</td>
<td>Pos</td>
<td>4</td>
</tr>
<tr>
<td>( f''' )</td>
<td>0</td>
<td>Pos</td>
<td>1</td>
<td>Pos</td>
<td>DNE</td>
<td>Pos</td>
<td>0</td>
</tr>
</tbody>
</table>

a) Find the absolute extrema of \( f \) and where they occur.

b) Find any points of inflection.

c) Sketch a possible graph of \( f \).