Ch 5 P2 Optimization Worksheet

1. Calculator Allowed] Wile E. is after Road Runner again! This time he’s got it figured out. Sitting on his ACME rocket he hides behind a hill anxiously awaiting the arrival that “beeping” bird. In his excitement to light the rocket he tips the rocket up. Instead of thrusting himself parallel to the ground where he can catch the Road Runner, he sends himself widely into the air following a path given by function
   \[ h(t) = 0.1t^3 - 1.3t^2 + 4.2t + 2, \]
   where \( h \) is the height of the rocket after \( t \) seconds. The rocket fuel lasts for 10 seconds. At that point, Wile E. Coyote stops suddenly and falls straight down to the ground.

   a) What is the domain of this function? \([0, 10]\)

   b) What is the highest point reached by Wile E. Coyote? [Use Calculus!]

   \[ h'(t) = 0.3t^2 - 2.6t + 4.2 \]
   \[ 0 = 0.3t^2 - 2.6t + 4.2 \]

   \[ t_1 = 2.148 \]
   \[ t_2 = 6.519 \]

\[ h(10) = 14 \]

Closed interval EVT

<table>
<thead>
<tr>
<th>( t )</th>
<th>( h(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2.148</td>
<td>6.015</td>
</tr>
<tr>
<td>6.519</td>
<td>1.837</td>
</tr>
<tr>
<td>10</td>
<td>14</td>
</tr>
</tbody>
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The highest point of 14 @ \( t = 10 \) sec

2. Find two positive numbers such that the product is 192 and the sum is a minimum.

\[ f(x) = x + y \]
\[ xy = 192 \]
\[ y = \frac{192}{x} \]

\[ f(x) = \frac{x + 192}{x} \]
\[ f'(x) = 1 - 192x^{-2} \]
\[ 0 = 1 - 192x^{-2} \]
\[ \frac{192}{x^2} = 1 \]
\[ \sqrt{x^2} = \sqrt{192} \]
\[ x = 8\sqrt{3} \]

\[ y = \frac{192}{8\sqrt{3}} \]
\[ y = \frac{24\sqrt{3}}{3} = 8\sqrt{3} \]
3. [No Calculator] A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$.

a) Draw a sketch the rectangle inscribed under the parabola.

b) Write a formula for the area of the inscribed rectangle as a function of $x$. What is the domain of this function?

$$A = l \cdot w, \quad A(x) = 2x(12 - x^2) \quad D: (0, 2\sqrt{3})$$

c) What is the largest area the rectangle can have, and what are its dimensions?

$$A(x) = 24x - 2x^3$$
$$A'(x) = 24 - 6x^2$$
$$0 = 24 - 6x^2$$
$$6x^2 = 24$$
$$x^2 = 4$$
$$x = \pm 2$$

4. [Calculator] The Profit $P$ in dollars made by a fast food restaurant selling $x$ hamburgers is given by

$$P = 2.44x - \frac{x^2}{20000}, \quad 0 \leq x \leq 35000.$$  

a) Find the intervals on which $P$ is increasing or decreasing. [Use Calculus!]

$$P' = 2.44 - \frac{x}{10000}$$
$$0 = 2.44 - \frac{x}{10000}$$
$$x = 24400$$

b) Find the maximum profit.

<table>
<thead>
<tr>
<th>Candidates</th>
<th>$x$</th>
<th>$P(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>-5000</td>
<td>-5000</td>
</tr>
<tr>
<td>24400</td>
<td>24748</td>
<td>24748</td>
</tr>
<tr>
<td>35000</td>
<td>19150</td>
<td>19150</td>
</tr>
</tbody>
</table>

$P(24400) = 24748$
5. A 216 - $m^2$ pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?

\[
A = 216 \text{m}^2
\]

\[
2xy = 216
\]

\[
y = \frac{216}{2x} = \frac{108}{x}
\]

\[
L = 3y + 4x
\]

\[
L(x) = 3\left(\frac{108}{x}\right) + 4x
\]

\[
L'(x) = -\frac{324}{x^2} + 4
\]

\[
0 = -\frac{324}{x^2} + 4
\]

\[
\frac{324}{x^2} = 4
\]

\[
x^2 = 81
\]

\[
x = 9
\]

Dimensions of outer rectangle:
18 x 12 m

72 m of fence needed

6. Find the length and width of a rectangle that has a perimeter of 64 feet and a maximum area.

\[
A(x) = xy
\]

\[
64 = 2x + 2y
\]

\[
y = 32 - x
\]

\[
A(x) = x(32-x)
\]

\[
A'(x) = 32 - 2x
\]

\[
0 = 32 - 2x
\]

\[
2x = 32
\]

\[
x = 16
\]

D: (0, 32)

A(16) = 16 x 16

\[
A(x) = 16 x 16
\]

when $x = 16$, $A(x)$ is maximized

[16 x 16]
7. Suppose you want to build a rectangular pen for your dog using a garage wall on one side and a fence on the other three sides. If you have 40 feet of fencing available, what should be the dimensions of the pen to yield the largest possible area?

\[ A(x) = xy \]
\[ 40 = 2x + y \]
\[ y = 40 - 2x \]
\[ A(x) = x(40-2x) \]
\[ A'(x) = 40 - 4x \]
\[ 0 = 40 - 4x \]
\[ 4x = 40 \]
\[ x = 10 \]

Dimensions: \(10 \times 20\)

8. A rectangular package to be sent by UPS can have a maximum combined length and girth of 300 cm. Find the dimensions of the package of maximum volume that can be sent [Assume that the cross section is a square.]

\[ V = x^2y \]
\[ y + 4x = 300 \]
\[ y = 300 - 4x \]

\[ V(x) = x^2(300 - 4x) \]
\[ V(x) = 300x^2 - 4x^3 \]
\[ V'(x) = 600x - 12x^2 \]
\[ 0 = 600x - 12x^2 \]
\[ 12x^2 - 600x = 0 \]
\[ 12x(x-50) = 0 \]
\[ x = 0, x = 50 \]

Dimensions: \(50 \times 50 \times 160\)
9. A rectangle is to be inscribed under one arch of the cosine curve from \([-\frac{\pi}{2}, \frac{\pi}{2}]\). What is the largest area the rectangle can have and what dimensions give that area?

\[ A(x) = 2x(\cos x) \]
\[ A'(x) = 2x(-\sin x) + 2(\cos x) \]
\[ 0 = -2x \sin x + 2 \cos x \]
\[ x = -0.86, \ x = 0.86 \]

When \( x = -0.86 \), \( A(x) \) is maximized

Largest Area: 1.12 \( \text{m}^2 \)
Dimensions: 1.73 \( \text{m} \times \frac{1}{2} \text{m} \)

10. A rectangle is bounded by the x-axis and the semicircle \( y = \sqrt{25 - x^2} \). What length and width should the rectangle have so that its area is a maximum?

\[ A(x) = 2x \left( \sqrt{25 - x^2} \right) \]
\[ A'(x) = 2x \left( \frac{1}{2} \left( \frac{1}{2} \sqrt{25 - x^2} \right) \right) (-2x) + 2 \sqrt{25 - x^2} \]
\[ = \frac{-4x^2}{\sqrt{25 - x^2}} + \frac{2 \sqrt{25 - x^2}}{\sqrt{25 - x^2}} \]
\[ = \frac{-4x^2 + 2 \sqrt{25 - x^2}}{\sqrt{25 - x^2}} \]
\[ = \frac{-2x^2 + 2(25 - x^2)}{\sqrt{25 - x^2}} \]
\[ = \frac{-2x^2 + 50 - 2x^2}{\sqrt{25 - x^2}} = \frac{-4x^2 + 50}{\sqrt{25 - x^2}} \]

When \( x = \frac{5\sqrt{2}}{2} \), \( A(x) \) is maximized

Width = \( 5\sqrt{2} \)