1. \[ \lim_{x \to 2} \frac{x^3 - 8}{x - 2} \]

2. \[ \lim_{h \to 0} \frac{\tan\left(\frac{\pi}{4} + h\right) - \tan\left(\frac{\pi}{4}\right)}{h} \]

3. Write the equation of the line tangent to the graph of \( y = 5x - \sin x \) at \( x = 2\pi \)

4. Let \( f(x) = \begin{cases} 9x - 4, & x \leq 1 \\ 4x^2 + 1, & x > 1 \end{cases} \)
Is this function continuous and/or differentiable at \( x = 1 \)

5. If \( \lim_{h \to 0} \frac{f(10 + h) - f(10)}{h} = 25 \), then which of the following may we assume to be true:

I \( f(10) = 25 \)
II \( f'(10) = 25 \)
III \( f \) is both continuous and differentiable at \( x = 10 \)

6. Given the following information about differentiable functions \( f(x) \) and \( g(x) \) at \( x = 2 \) determine the following:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
<th>( f'(x) )</th>
<th>( g'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8</td>
<td>2</td>
<td>2\pi</td>
<td>( e )</td>
</tr>
</tbody>
</table>

(a) \[ \frac{d}{dx} \left[ f(g(x)) \right] \text{ at } x = 2 \]

(b) \[ \frac{d}{dx} \left( \frac{1}{f(x)} \right) \text{ at } x = 2 \]

(c) \[ \frac{d}{dx} \left[ \frac{g(x)}{f(x)} \right] \text{ at } x = 2 \]

7. Find \[ \frac{d}{dx} \left[ \sin \sqrt[3]{x} \right] \]

8. Find \( f'(x) \) if \( f(x) = \frac{2x + 3}{3x + 2} \)
9. Given \(25x^2 + 8x - 16y^2 - 4y - 9 = 0\)
   (a) Find \(\frac{dy}{dx}\)
   (b) Find any value(s) of \(x\) where the curve has a horizontal tangent
   (c) Find the value(s) of \(y\) where the curve has a vertical tangent

10. What is the slope of the tangent line to the graph of \(y = \tan(2x)\) at \(x = \frac{\pi}{8}\)?

Some calculator-friendly problems:

11. A particle moves along the x-axis so that any time \(t \geq 0\), its velocity is given by
    \(v(t) = t^3 \sin t\). Find the acceleration at \(t = 3\) AND determine if the speed is increasing or decreasing at \(t = 3\)

12. A pebble is thrown into a pond forming ripples whose radius increases at a rate of 4 inches/second. How fast is the area of the ripple changing when the radius is 12 inches?

13. On the interval \([0, \pi]\), where do the graphs of \(f(x) = \tan x\) and \(g(x) = x^2\) have parallel tangent lines?

14. The position \(s(t)\) of a particle is measured every 10 seconds and is provided in the table below.

<table>
<thead>
<tr>
<th>(t) [in seconds]</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s(t)) [in feet]</td>
<td>0</td>
<td>12</td>
<td>15</td>
<td>17</td>
<td>30</td>
<td>60</td>
</tr>
</tbody>
</table>

   (A) Estimate the instantaneous velocity of the particle at \(t = 25\) seconds. Include units.

   (B) Find the average velocity for the time interval \([0, 50]\)

15. Find the following:
   (A) \(\frac{d}{dx} [7 \cos^3(\pi x)]\)
   (B) \(\frac{d}{dt} \left[ \frac{\pi}{3} r^2 h \right]\)
16.

**2008 AP\textsuperscript{c} CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

6. Consider the closed curve in the xy-plane given by

\[ x^2 + 2x + y^4 + 4y = 5. \]

(a) Show that \( \frac{dy}{dx} = \frac{-(x + 1)}{2(y^3 + 1)} \).

(b) Write an equation for the line tangent to the curve at the point \((-2, 1)\).

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.

17.

The graph above is the velocity graph of a particle moving along the x-axis.

(A) When is the particle at rest? Justify.

(B) When does the particle change direction? Justify.

(C) What is the acceleration at time, \( t = 2 \).

Be sure to remember how to do anything that we have done on any projects, puzzles, or quizzes for this chapter. [I want to save paper!]

Revised on 30 September 2009