AP REVIEW SESSION 6

Differential Equations & Modeling

- Derivatives of Inverse Functions
- Differential Equations
- Exponential Growth & Decay
- Slope Fields
- Definite/Indefinite Integration with U-substitution

No-Calc

2003

2. \[ \int_0^1 e^{-4x} \, dx = \]
   (A) \( \frac{-e^{-4}}{4} \)  (B) \(-4e^{-4}\)  (C) \( e^{-4} - 1 \)  (D) \( \frac{1}{4} - \frac{e^{-4}}{4} \)  (E) \( 4 - 4e^{-4} \)

8. \[ \int x^2 \cos(x^3) \, dx = \]
   (A) \( -\frac{1}{3} \sin(x^3) + C \)
   (B) \( \frac{1}{3} \sin(x^3) + C \)
   (C) \( -\frac{x^3}{3} \sin(x^3) + C \)
   (D) \( \frac{x^3}{3} \sin(x^3) + C \)
   (E) \( \frac{x^3}{3} \sin\left(\frac{x^4}{4}\right) + C \)
11. Using the substitution \( u = 2x + 1 \), \( \int_0^2 \sqrt{2x + 1} \, dx \) is equivalent to

(A) \( \frac{1}{2} \int_{1/2}^{r/2} \sqrt{u} \, du \) \hspace{1cm} (B) \( \frac{1}{2} \int_0^2 \sqrt{u} \, du \) \hspace{1cm} (C) \( \frac{1}{2} \int_1^5 \frac{\sqrt{u}}{u} \, du \) \hspace{1cm} (D) \( \int_0^2 \sqrt{u} \, du \) \hspace{1cm} (E) \( \int_0^5 \frac{\sqrt{u}}{u} \, du \)

12. The rate of change of the volume, \( V \), of water in a tank with respect to time, \( t \), is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

(A) \( V(t) = k\sqrt{t} \)
(B) \( V(t) = k\sqrt{V} \)
(C) \( \frac{dV}{dt} = k\sqrt{t} \)
(D) \( \frac{dV}{dt} = \frac{k}{\sqrt{V}} \)
(E) \( \frac{dV}{dt} = k\sqrt{V} \)

19. A curve has slope \( 2x + 3 \) at each point \((x, y)\) on the curve. Which of the following is an equation for this curve if it passes through the point \((1, 2)\)?

(A) \( y = 5x - 3 \)
(B) \( y = x^2 + 1 \)
(C) \( y = x^2 + 3x \)
(D) \( y = x^2 + 3x - 2 \)
(E) \( y = 2x^2 + 3x - 3 \)
27. Let \( f \) be the function defined by \( f(x) = x^3 + x \). If \( g(x) = f^{-1}(x) \) and \( g(2) = 1 \), what is the value of \( g'(2) \)?

(A) \( \frac{1}{13} \)  
(B) \( \frac{1}{4} \)  
(C) \( \frac{7}{4} \)  
(D) 4  
(E) 13

1. \( \int \cos(3x) \, dx = \)

(A) \(-3\sin(3x) + C\)  
(B) \(\frac{1}{3}\sin(3x) + C\)  
(C) \(\frac{1}{3}\sin(3x) + C\)  
(D) \(\sin(3x) + C\)  
(E) \(3\sin(3x) + C\)

7. \( \int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx = \)

(A) \(2e^{\sqrt{x}} + C\)  
(B) \(\frac{1}{2}e^{\sqrt{x}} + C\)  
(C) \(e^{\sqrt{x}} + C\)  
(D) \(2\sqrt{x}e^{\sqrt{x}} + C\)  
(E) \(\frac{1}{2}e^{\sqrt{x}} + C\)
13. \[ \int (x^3 + 1)^2 \, dx = \]

(A) \( \frac{1}{7} x^7 + x + C \)

(B) \( \frac{1}{7} x^7 + \frac{1}{2} x^4 + x + C \)

(C) \( 6x^2 (x^3 + 1) + C \)

(D) \( \frac{1}{3} (x^3 + 1)^3 + C \)

(E) \( \frac{(x^3 + 1)^3}{9x^2} + C \)

15. The slope field for a certain differential equation is shown above. Which of the following could be a solution to the differential equation with the initial condition \( y(0) = 1 \) ?

(A) \( y = \cos x \)

(B) \( y = 1 - x^2 \)

(C) \( y = e^x \)

(D) \( y = \sqrt{1 - x^2} \)

(E) \( y = \frac{1}{1 + x^2} \)
28. If \( y = \sin^{-1}(5x) \), then \( \frac{dy}{dx} = \)

(A) \( \frac{1}{1 + 25x^2} \)

(B) \( \frac{5}{1 + 25x^2} \)

(C) \( \frac{-5}{\sqrt{1 - 25x^2}} \)

(D) \( \frac{1}{\sqrt{1 - 25x^2}} \)

(E) \( \frac{5}{\sqrt{1 - 25x^2}} \)

**Calculator Allowed**

2008

90. The functions \( f \) and \( g \) are differentiable. For all \( x \), \( f(g(x)) = x \) and \( g(f(x)) = x \).

If \( f'(3) = 8 \) and \( f'(3) = 9 \), what are the values of \( g(8) \) and \( g'(8) \)?

(A) \( g(8) = \frac{1}{3} \) and \( g'(8) = -\frac{1}{9} \)

(B) \( g(8) = \frac{1}{3} \) and \( g'(8) = \frac{1}{9} \)

(C) \( g(8) = 3 \) and \( g'(8) = -9 \)

(D) \( g(8) = 3 \) and \( g'(8) = -\frac{1}{9} \)

(E) \( g(8) = 3 \) and \( g'(8) = \frac{1}{9} \)
5. Consider the differential equation $\frac{dy}{dx} = \frac{x}{y}$, where $y \neq 0$.

(a) The slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point (3, −1), and sketch the solution curve that passes through the point (1, 2).

(Note: The points (3, −1) and (1, 2) are indicated in the figure.)

(b) Write an equation for the line tangent to the solution curve that passes through the point (1, 2).

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(3) = −1$, and state its domain.