AP REVIEW SESSION 3

More Derivatives

- Chain Rule for Differentiation
- Related Rates
- Implicit Differentiation
- Derivatives of non-polynomial functions.

Non Calc

2003

1. If \( y = (x^3 + 1)^2 \), then \( \frac{dy}{dx} = \)
   
   (A) \( (3x^2)^2 \) \quad (B) \( 2(x^3 + 1) \) \quad (C) \( 2(3x^2 + 1) \) \quad (D) \( 3x^2(x^3 + 1) \) \quad (E) \( 6x^2(x^3 + 1) \)

9. If \( f(x) = \ln(x + 4 + e^{-2x}) \), then \( f'(0) \) is
   
   (A) \( -\frac{2}{3} \) \quad (B) \( \frac{1}{3} \) \quad (C) \( \frac{1}{4} \) \quad (D) \( \frac{2}{3} \) \quad (E) nonexistent

14. If \( y = x^2 \sin 2x \), then \( \frac{dy}{dx} = \)
   
   (A) \( 2x \cos 2x \)
   (B) \( 4x \cos 2x \)
   (C) \( 2x(\sin 2x + \cos 2x) \)
   (D) \( 2x(\sin 2x - x \cos 2x) \)
   (E) \( 2x(\sin 2x + x \cos 2x) \)
26. What is the slope of the line tangent to the curve \( 3y^2 - 2x^2 = 6 - 2xy \) at the point \((3, 2)\)?

(A) 0  (B) \( \frac{4}{9} \)  (C) \( \frac{7}{9} \)  (D) \( \frac{6}{7} \)  (E) \( \frac{5}{3} \)

2008

4. If \( f(x) = \cos^3(4x) \), then \( f'(x) = \)

(A) \( 3\cos^2(4x) \)
(B) \( -12\cos^2(4x)\sin(4x) \)
(C) \( -3\cos^2(4x)\sin(4x) \)
(D) \( 12\cos^2(4x)\sin(4x) \)
(E) \( -4\sin^3(4x) \)

6. Let \( f \) be the function given by \( f(x) = (2x - 1)^5(x + 1) \). Which of the following is an equation for the line tangent to the graph of \( f \) at the point where \( x = 1 \)?

(A) \( y = 21x + 2 \)
(B) \( y = 21x - 19 \)
(C) \( y = 11x - 9 \)
(D) \( y = 10x + 2 \)
(E) \( y = 10x - 8 \)
24. The radius of a circle is increasing. At a certain instant, the rate of increase in the area of the circle is numerically equal to twice the rate of increase in its circumference. What is the radius of the circle at that instant?

(A) \(\frac{1}{2}\)  
(B) 1  
(C) \(\sqrt{2}\)  
(D) 2  
(E) 4

25. If \(x^2y - 3x = y^3 - 3\), then at the point \((-1, 2)\), \(\frac{dy}{dx} = \)

(A) \(-\frac{7}{11}\)  
(B) \(-\frac{7}{13}\)  
(C) \(-\frac{1}{2}\)  
(D) \(-\frac{3}{14}\)  
(E) 7

**Calculator Allowed**

2003

78. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is \(20\pi\) meters?

(A) 0.04\(\pi\) m\(^2\)/sec
(B) 0.4\(\pi\) m\(^2\)/sec
(C) 4\(\pi\) m\(^2\)/sec
(D) 20\(\pi\) m\(^2\)/sec
(E) 100\(\pi\) m\(^2\)/sec
5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let \( h \) be the depth of the coffee in the pot, measured in inches, where \( h \) is a function of time \( t \), measured in seconds. The volume \( V \) of coffee in the pot is changing at the rate of \(-5\pi \sqrt{h}\) cubic inches per second. (The volume \( V \) of a cylinder with radius \( r \) and height \( h \) is \( V = \pi r^2 h \).)

(a) Show that \( \frac{dh}{dt} = -\frac{\sqrt{h}}{5} \).

(b) Given that \( h = 17 \) at time \( t = 0 \), solve the differential equation \( \frac{dh}{dt} = -\frac{\sqrt{h}}{5} \) for \( h \) as a function of \( t \).

(c) At what time \( t \) is the coffeepot empty?