Sample Examination III

Section I Part A

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. Calculators may NOT be used on this part of the exam.

In this exam:

(1) Unless otherwise specified, the domain of a function \( f \) is assumed to be the set of all real numbers \( x \) for which \( f(x) \) is a real number.

(2) The inverse of a trigonometric function \( f \) may be indicated using the inverse function notation \( f^{-1} \) or with the prefix "arc" (e.g., \( \sin^{-1} x = \arcsin x \))

1. The area of the region between the graph of \( y = 3x^2 + 2x \) and the \( x \)-axis from \( x = 1 \) to \( x = 3 \) is

(A) 36   (B) 34   (C) 31   (D) 26   (E) 12

Answer

\[
\begin{align*}
  f(x) & = \begin{cases} 
    x^2 & \text{for } x \neq 0 \\
    \frac{x}{|x|} & \text{for } x = 0 
  \end{cases} 
\end{align*}
\]

2. Let \( f \) be the continuous function defined above. What is the value of \( \int_{-4}^{2} f(x) \, dx \)?

(A) -10   (B) -6   (C) 2   (D) 6   (E) 10

Answer
3. For a car traveling at a speed of \( s \) miles per hour, the fuel consumption of the car, \( C(s) \), is measured in gallons per mile. What are the units of \( \int_a^b C(s) \, ds \)?

(A) gallons
(B) hours per gallon
(C) gallons per hour
(D) miles per hour per gallon
(E) gallons per miles per hour

Answer

4. If the radius of a sphere is increasing at the rate of 2 inches per second, how fast, in cubic inches per second, is the volume increasing when the radius is 10 inches?

(The volume of a sphere with radius \( r \) is \( V = \frac{4}{3}\pi r^3 \).)

(A) \( 40\pi \)  (B) \( 80\pi \)  (C) \( 400\pi \)  (D) \( 800\pi \)  (E) \( 3200\pi \)

Answer
5. A particle moves along a straight line so that its velocity is given by \( v(t) = t^2 \). How far does the particle travel between \( t = 1 \) and \( t = 3 \)?

(A) \( \frac{1}{3} \)  (B) \( \frac{26}{3} \)  (C) 8  (D) 26  (E) 27

6. The table above gives values of a function \( f \) and its derivative at selected values of \( x \). If \( f' \) is continuous on \([0,5]\), what is the value of \( \int_{1}^{4} f'(x) \, dx \)?

(A) -5  (B) -4  (C) 2  (D) 7  (E) 9
7. The slope field for a differential equation \( \frac{dy}{dx} = f(y) \) is shown in the figure above.

Which statement is true about \( y(x) \)?

I. If \( y(0) > 2 \), then \( \lim_{x \to \infty} y(x) \approx 2 \).

II. If \( 0 < y(0) < 2 \), then \( \lim_{x \to \infty} y(x) \approx 2 \).

III. If \( y(0) < 0 \), then \( \lim_{x \to \infty} y(x) \approx 2 \).

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

Answer
8. \[ \int_{1}^{3} \frac{x}{x^2 + 1} \, dx = \]

(A) \( \ln 5 \)  (B) \( \ln 10 \)  (C) \( 2 \ln 5 \)  (D) \( \frac{1}{2} \ln 5 \)  (E) \( \ln \left( \frac{5}{2} \right) \)

---

9. \[ \frac{d}{dx} \ln \left( \frac{1}{x^2 - 1} \right) = \]

(A) \( \frac{1}{x^2 - 1} \)  
(B) \( \frac{2x}{1 - x^2} \)  
(C) \( \frac{2x}{x^2 - 1} \)  
(D) \( 2x^3 - 2x \)  
(E) \( 2x - 2x^3 \)
10. For $0 \leq x < \frac{\pi}{2}$, an antiderivative of $2 \tan x$ is

(A) $\ln (\sec 2x)$
(B) $2 \sec^2 x$
(C) $\ln (\sec^2 x)$
(D) $2 \ln (\cos x)$
(E) $\ln (2 \sec x)$

11. A function $f$ is continuous on the closed interval $[4,6]$ and twice differentiable on the open interval $(4,6)$. If $f'(5) = -3$, and $f$ is concave downwards on the given interval, which of the following could be a table of values for $f$?

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<thead>
<tr>
<th>(A) $x$</th>
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<tbody>
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<td>4</td>
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<td>5</td>
<td>4</td>
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<th>(D) $x$</th>
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<td>5</td>
<td>3</td>
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<tr>
<td>6</td>
<td>2</td>
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</table>

Answer

Answer
12. The equation of the line tangent to the curve $y = \frac{kx + 8}{k + x}$ at $x = -2$ is $y = x + 4$. What is the value of $k$?

(A) $-3$   (B) $-1$   (C) $1$   (D) $3$   (E) $4$

13. A function $f$ is continuous on the closed interval $[5, 12]$ and differentiable on the open interval $(5, 12)$ and $f$ has the values given in the table above. Using the subintervals $[5, 6], [6, 9], [9, 11]$, and $[11, 12]$, what is the right Riemann sum approximation to $\int_{5}^{12} f(x) \, dx$?

(A) $64$   (B) $65$   (C) $66$   (D) $68.5$   (E) $72$
14. A function \( f(x) \) has a vertical asymptote at \( x = 2 \). Which of the following statements are true?

I. \( \lim_{x \to 2^-} f(x) = +\infty \)

II. \( \lim_{x \to 2^+} f(x) = +\infty \)

III. \( \lim_{x \to 2^-} f(x) = +\infty \)

(A) I only  
(B) II only  
(C) III only  
(D) I and II only  
(E) I, II, and III

Answer

15. The following statements concerning the location of an extreme value of a twice-differentiable function, \( f \), are all true. Which statement also includes the correct justification?

(A) The function has a maximum at \( x = 5 \) because \( f'(5) = 0 \).

(B) The function has a maximum at \( x = 5 \) because \( f'(x) < 0 \) for \( x < 5 \) and \( f'(x) > 0 \) for \( x > 5 \).

(C) The function has a minimum at \( x = 3 \) because the tangent line at \( x = 3 \) is horizontal.

(D) The function has a minimum at \( x = 3 \) because \( f'(x) < 0 \) for \( x < 3 \) and \( f'(x) > 0 \) for \( x > 3 \).

(E) The function has a minimum at \( x = 3 \) because \( f''(3) < 0 \).

Answer
16. The graph of a differentiable function $f$ is shown above. The graph has a relative minimum at $x = 1$ and a relative maximum at $x = 5$. Let $g$ be the function defined by $g(x) = \int_0^x f(t) \, dt$. For what value of $x$ does the graph of $g$ change from concave up to concave down?

(A) 0   (B) 1   (C) 2   (D) 5   (E) 7
17. The graph of the derivative of \( f \) is shown in the figure above. Which of the following could be the graph of \( f \)?

(A) \( y \) \( x \)

(B) \( y \) \( x \)

(C) \( y \) \( x \)

(D) \( y \) \( x \)

(E) \( y \) \( x \)

Answer: [Blank]
18. \( \frac{d}{dx} \int_{x}^{0} \frac{du}{1 + u^2} = \)

(A) \( \frac{1}{x^2 + 1} \)  
(B) \( \frac{-1}{x^2 + 1} \)  
(C) \( x^2 + 1 \)  
(D) \( -x^2 + 1 \)  
(E) \( \tan^{-1}(x) \)

19. Let \( f(x) \) be a differentiable function defined only on the interval \( -2 \leq x \leq 10 \). The table below gives the value of \( f(x) \) and its derivative \( f'(x) \) at several points of the domain.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>26</td>
<td>27</td>
<td>26</td>
<td>23</td>
<td>18</td>
<td>11</td>
<td>2</td>
</tr>
<tr>
<td>( f'(x) )</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td>-5</td>
</tr>
</tbody>
</table>

The line tangent to the graph of \( f(x) \) and parallel to the segment between the endpoints of \( f(x) \) intersects the y-axis at the point

(A) (0, 27)  
(B) (0, 28)  
(C) (0, 31)  
(D) (0, 36)  
(E) (0, 43)
20. At each point \((x, y)\) on a certain curve, the slope of the curve is \(4xy\). If the curve contains the point \((0, 4)\), then its equation is

(A) \(y = e^{2x^2} + 4\)
(B) \(y = e^{2x^2} + 3\)
(C) \(y = 4e^{2x^2}\)
(D) \(y^2 = 2x^2 + 4\)
(E) \(y = 2x^2 + 4\)

Answer

21. An equation of the line tangent to the curve \(y = x^3 - 6x^2\) at its point of inflection is

(A) \(y = -12x + 8\)
(B) \(y = -12x + 40\)
(C) \(y = 12x - 8\)
(D) \(y = -12x + 12\)
(E) \(y = 12x - 40\)

Answer
22. If $0 < k < \pi$, then $\int_0^k \cos(2x) \, dx = \frac{1}{2}$ when $k =$

(A) $\frac{\pi}{12}$  (B) $\frac{\pi}{4}$  (C) $\frac{5\pi}{12}$  (D) $\frac{\pi}{2}$  (E) $\frac{3\pi}{4}$

23. Which of the following are properties of the definite integral?

I. $\int_a^b k f(x) \, dx = k \int_a^b f(x) \, dx$, $k \neq 0$

II. $\int_a^b x f(x) \, dx = x \int_a^b f(x) \, dx$

III. $\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx$

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II, and III
24. If \( f(x) = \sqrt{e^{2x}} + 1 \), then \( f'(0) = \)

(A) \( \frac{-\sqrt{2}}{2} \)  \hspace{1cm} (B) \( \frac{\sqrt{2}}{4} \)  \hspace{1cm} (C) \( \frac{\sqrt{2}}{2} \)  \hspace{1cm} (D) 1  \hspace{1cm} (E) \( \sqrt{2} \)

25. If \( x^2y + yx^2 = 6 \), then \( \frac{d^2y}{dx^2} \) at the point \((1, 3)\) is

(A) \(-18\)  \hspace{1cm} (B) \(-6\)  \hspace{1cm} (C) \(6\)  \hspace{1cm} (D) \(12\)  \hspace{1cm} (E) \(18\)
26. The base of a solid is the region in the first quadrant bounded by the line \( x + 2y = 4 \) and the coordinate axes. What is the volume of the solid if every cross section perpendicular to the \( x \)-axis is a semicircle?

(A) \( \frac{2\pi}{3} \)  
(B) \( \frac{4\pi}{3} \)  
(C) \( \frac{8\pi}{3} \)  
(D) \( \frac{32\pi}{3} \)  
(E) \( \frac{64\pi}{3} \)

27. A particle moves along the \( x \)-axis so that at any time \( t \) its position is given by \( x(t) = (t + 1)(t - 3)^3 \). For what values of \( t \) is the velocity of the particle increasing?

(A) \( t > 3 \) only

(B) \( 0 < t < 3 \) only

(C) \( 1 < t < 3 \) only

(D) \( t < 1 \) or \( t > 3 \)

(E) \( 0 < t < 3 \) or \( t > 3 \)
28. At any time $t \geq 0$, in days, the rate of growth of a bacteria population is given by $y' = ky$, where $y$ is the number of bacteria present and $k$ is a constant. The initial population is 1,500 and the population is quadrupled during the first 2 days. By what factor will the population have increased during the first 3 days?

(A) 4 (B) 5 (C) 6 (D) 8 (E) 10
Section I Part B

Directions: Solve each of the following problems, using available space for scratch work. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. Calculators may be used on this part of the exam.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function \( f \) is assumed to be the set of all real numbers \( x \) for which \( f(x) \) is a real number.

(3) The inverse of a trigonometric function \( f \) may be indicated using the inverse function notation \( f^{-1} \) or with the prefix "arc" (e.g., \( \sin^{-1}x = \arcsin x \))

29. The average value of a continuous function \( f(x) \) on the closed interval \([3, 7]\) is 12.

What is the value of \( \int_{3}^{7} f(x) \, dx \)?

(A) 3  (B) 4  (C) 12  (D) 36  (E) 48
30. The region in the first quadrant enclosed by the coordinate axes, the line \( x = \pi \), and the curve \( y = \cos(\cos x) \) is rotated about the \( x \)-axis. What is the volume of the solid generated?

(A) 1.922 \hspace{1cm} (B) 3.782 \hspace{1cm} (C) 6.040 \hspace{1cm} (D) 8.128 \hspace{1cm} (E) 9.867

31. The functions \( f \) and \( g \) are piecewise linear functions whose graphs are shown above.

If \( h(x) = \frac{f(x)}{g(x)} \), then \( h'(3) = \)

(A) \(-\frac{2}{9}\) \hspace{1cm} (B) \(-\frac{1}{3}\) \hspace{1cm} (C) 0 \hspace{1cm} (D) \(\frac{1}{3}\) \hspace{1cm} (E) \(\frac{8}{27}\)
32. The region bounded by the x-axis and the part of the graph of \( y = \cos x \) between \( x = 0 \) and \( x = \pi/2 \) is divided into two regions by the line \( x = c \). If the area of the region for \( 0 \leq x \leq c \) is equal to the area of the region for \( c \leq x \leq \pi/2 \), then \( c \) must be

(A) \( \frac{\pi}{4} \)  (B) \( \frac{\pi}{6} \)  (C) \( \frac{\pi}{3} \)  (D) \( \frac{5\pi}{36} \)  (E) \( \frac{7\pi}{36} \)

Answer

33. For what values of \( x \) is the function \( f(x) = 5 + 15x + 6x^2 - x^3 \) decreasing?

(A) \(-1 < x < 5\)
(B) \(-5 < x < 1\)
(C) \(x < -5 \) or \( x > 1\)
(D) \(x < -1 \) or \( x > 5\)
(E) All real numbers \( x \)

Answer
34. If $f$ is an antiderivative of $\frac{\tan^2 x}{x^2 + 1}$ such that $f(1) = \frac{1}{2}$, then $f(0) =$

(A) 0  (B) 0.155  (C) 0.345  (D) 0.845  (E) 1

35. The second derivative of a function is given by $f''(x) = 0.5 + \cos x - e^{-x}$. How many points of inflection does the function $f(x)$ have on the interval $0 \leq x \leq 20$?

(A) None  (B) Three  (C) Six  (D) Seven  (E) Ten
36. The local linear approximation of a function $f$ will always be greater than or equal to the function’s value if, for all $x$ in an interval containing the point of tangency,

(A) $f’ < 0$  (B) $f’ > 0$  (C) $f'' < 0$  (D) $f'' > 0$  (E) $f’ = f'' = 0$

37. Let $f$ be the function given by $f(x) = 5 + 5.8 \sin \left( \frac{\pi x}{4} \right) - 15.7 \cos \left( \frac{\pi x}{3} \right)$.

For $0 \leq x \leq 12$, $f$ is increasing most rapidly when $x$ equals

(A) 1.328  (B) 4.434  (C) 6.000  (D) 7.566  (E) 10.672
38. If \( \lim_{x \to 3} \frac{g(3) - g(x)}{3 - x} = -0.628 \), then near the point where \( x = 3 \), the graph of \( g(x) \)

(A) is decreasing
(B) is increasing
(C) is concave upwards
(D) is concave downwards
(E) has a point of inflection

Answer

39. How many relative extreme values does the function whose derivative is given by \( y' = \sin(\ln x) \) have in the interval \( 0 < x \leq 1 \)?

(A) One  (B) Two  (C) Three  (D) Four  (E) More than four

Answer
40. Let \( f(x) = x^3 - 7x^2 + 25x - 39 \) and let \( g \) be the inverse function of \( f \). What is the value of \( g'(0) \)?

(A) \(-\frac{1}{25}\)  (B) \(\frac{1}{25}\)  (C) \(\frac{1}{10}\)  (D) 10  (E) 25

41. The expression \( 4\int_{0}^{2} \rho(x) \, dx \) gives the number of people living on one side of a 4-mile long stretch of highway, where \( x \) is the number of miles from the highway. What are the units of \( \rho(x) \)?

(A) Miles  
(B) People  
(C) Square miles  
(D) People per mile  
(E) People per square mile
42. A rectangle inscribed in a semicircle of radius 8 has one side lying on the diameter of the circle. What is the maximum possible area of the rectangle?

(A) $4\sqrt{2}$  (B) $8\sqrt{2}$  (C) 32  (D) $32\sqrt{2}$  (E) 64

43. Let $f$ be the function defined above, where $a$ and $b$ are constants. If $f$ is differentiable at $x = 0$, what is the value of $a + b$?

(A) $-4$  (B) $-2$  (C) 0  (D) 2  (E) 4
44. Silt is being dredged out of a river bed at the rate given by \[ R(t) = 100 \left( \frac{5t^2 - t - 1}{5t^2 + t} \right) \text{ gallons per minute, where} \ t \text{ is measured in minutes. Approximately how much silt is dredged from the river bed between} \ t = 2 \text{ to} \ t = 10 \text{ minutes?} \]

(A) 18.6 gallons  
(B) 95.8 gallons  
(C) 731 gallons  
(D) 753 gallons  
(E) 798 gallons

Answer

45. The line \( y = mx + b \) with \( b \geq 2 \) is tangent to the graph of \( f(x) = -2(x - 2)^2 + 2 \) at a point in the first quadrant. What are all possible values of \( b \) ?

(A) \( b = 2 \) only  
(B) \( 2 \leq b < 10 \)  
(C) \( 2 \leq b < 12 \)  
(D) \( 2 \leq b < 14 \)  
(E) \( 2 \leq b < 20 \)

Answer
SECTION II - FREE-RESPONSE QUESTIONS
GENERAL INSTRUCTIONS

For each part of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM IN THE SPACE PROVIDED FOR THAT PART. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.

- Show all your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit.

- Justifications require that you give mathematical (noncalculator) reasons.

- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, \( \int_1^5 x^2 \, dx \) may not be written as fnInt(X^2, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

- If you use decimal approximations in calculations, your work will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function \( f \) is assumed to be the set of all real numbers \( x \) for which \( f(x) \) is a real number.

SECTION II PART A: 30 Minutes, Questions 1,2

A graphing calculator is required.

During the timed portion for Part A, you may work only on the problems in Part A. Write your solution to each part of each problem in the space provided.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your programs, you must show the mathematical steps necessary to produce your results.

Do not go on to Part B until you are told to do so.

SECTION II PART B: 60 Minutes, Questions 3,4,5,6

Write your solution to each part of each problem in the space provided for that part. During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.
1. A particle is moving along the y-axis so that at any time \( t \) its velocity is given by

\[ v(t) = (t - 4)^4 \sin(t - 4) \]

(a) Find the velocity, speed, and acceleration of the particle at \( t = 3 \).

(b) In the viewing window provided below, sketch the graph of \( v(t) \).

(c) In the interval \( 3 \leq t \leq 5 \) is the velocity increasing or decreasing? Use your graph to justify your answer.

(d) Find the distance the particle travels from \( t = 3 \) to \( t = 5 \). Show how you arrived at your answer.

(a) Find the velocity, speed, and acceleration of the particle at \( t = 3 \).
(b) In the viewing window provided below, sketch the graph of $v(t)$.

(c) In the interval $3 \leq t \leq 5$ is the velocity increasing or decreasing? Use your graph to justify your answer.

(d) Find the distance the particle travels from $t = 3$ to $t = 5$. Show how you arrived at your answer.
2. Sand is being poured into a bin that is initially empty. During the work day, for $0 \leq t \leq 9$ hours, the sand pours into the bin at the rate given by

$$S(t) = \frac{5000}{t^3 + 50} \text{ cubic meters per hour.}$$

After one hour, for $1 \leq t \leq 9$, sand is removed from the bin at the rate of

$$R(t) = 23.9665 \sqrt{t} \text{ cubic meters per hour.}$$

(a) How much sand is poured into the bin during the work day? Include units of measure.

(b) Find $S(6) - R(6)$ and include units of measure. Explain what this amount means in the context of the problem.

(c) Explain why the amount of sand in the bin is at a maximum when $S(t) = R(t)$.

(d) How much sand, in cubic meters, is in the bin at the end of the work day?

---

(a) How much sand is poured into the bin during the work day? Include units of measure.

(b) Find $S(6) - R(6)$ and include units of measure. Explain what this amount means in the context of the problem.
(c) Explain why the amount of sand in the bin is at a maximum when \( S(t) = R(t) \).

(d) How much sand, in cubic meters, is in the bin at the end of the work day?
3. Let \( R \) be the region bounded by the graphs of \( y = \sqrt{x} \) and \( y = \frac{x}{2} \) as shown in the figure above.

(a) Find the area of \( R \).

(b) Find the volume of the solid generated when \( R \) is revolved around the y-axis.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when \( R \) is revolved around the horizontal line \( y = -3 \).

---

(a) Find the area of \( R \).
(b) Find the volume of the solid generated when $R$ is revolved around the $y$-axis.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when $R$ is revolved around the horizontal line $y = -3$. 
4. The region, \( R \), is bounded by the graphs of \( f(x) = x^2 - 3 \), \( g(x) = (x - 3)^2 \), and the line, \( T \), as shown in the figure above. \( T \) is tangent to the graph of \( f \) at the point \( (a, a^2 - 3) \) and tangent to the graph of \( g \) at the point \( (b, (b - 3)^2) \).

(a) Show that \( a = b - 3 \).

(b) Find the numerical value of \( a \) and the numerical value of \( b \).

(c) Write an equation of the line \( T \).

(d) Set up, but do not evaluate, an integral expression that gives the area of region \( R \).

\[ \text{(a) Show that } a = b - 3. \]
(b) Find the numerical value of $a$ and the numerical value of $b$.

(c) Write an equation of the line $T$.

(d) Set up, but do not evaluate, an integral expression that gives the area of region $R$. 
5. Let \( f(x) \) be the function defined by \( f(x) = k + 12x + 3x^2 - 2x^3 \), where \( k \) is a constant.

(a) On what interval is the function increasing? Justify your answer.

(b) If the relative maximum value of \( f \) is 4, what is the value of \( k \)?

(c) Find the interval where the function is concave up. Justify your answer.

(d) Find the relative minimum value of the function.

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(a) On what interval is the function increasing? Justify your answer.

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(b) If the relative maximum value of \( f \) is 4, what is the value of \( k \)?
(c) Find the interval where the function is concave up. Justify your answer.

(d) Find the relative minimum value of the function.
6. Let \( f \) be a function defined on the closed interval \([-3, 9]\). The graph of \( f \), consisting of three line segments is shown above. Let \( g(x) = \int_{0}^{x} f(t) \, dt \).

(a) Find \( g(4.5), g'(4.5) \), and \( g''(4.5) \).

(b) Find the average value of \( f \) on the closed interval \([-3, 5]\). Show the work that leads to your answer.

(c) Find the \( x \)-coordinate of any points of inflection of \( g \). Justify your answer.

(d) Find the coordinates of all maximum points of \( g \).

(a) Find \( g(4.5), g'(4.5) \), and \( g''(4.5) \).
(b) Find the average value of $f$ on the closed interval $[-3,5]$. Show the work that leads to your answer.

(c) Find the $x$-coordinate of any points of inflection of $g$. Justify your answer.

(d) Find the coordinates of all maximum points of $g$. 
1. a. speed = 0.841, velocity = −0.841, acceleration = 3.906  
b. [Graph: increasing]  
c. increasing  
d. 0.293

2. a. 415.420 or 415.421 cubic meters.  
b. −39.908 or −39.909 cubic meters per hour. This means that the amount of sand in 
the bin is decreasing at the rate of 39.908 (or 39.909) cubic meters per hour when \( t = 6 \).  
c. The amount of sand, \( A(t) \), in 
the bin is given by \( A(t) = \int_0^t S(x) \, dx - \int_1^t R(x) \, dx \). The maximum occurs when \( A'(t) = S(t) - R(t) = 0 \) or when \( S(t) = R(t) \).  
d. 0.00158 or 0.002 cubic meters.

3. a. \( \frac{4}{3} \)  
b. \( \frac{64\pi}{15} \)  
c. \( \pi \int_0^4 \left( (\sqrt{x} - (-3))^2 - \left( \frac{x}{2} - (-3) \right)^2 \right) \, dx \)

4. a. see solutions manual  
b. \( a = \frac{1}{2} \) and \( b = \frac{7}{2} \)  
c. \( y - \left( \left( \frac{1}{2} \right)^2 - 3 \right) = 1 \left( x - \frac{1}{2} \right) \) or \( y = x - \frac{13}{4} \)  
d. \( \int_{\frac{1}{2}}^2 \left( x^2 - 3 - (x - \frac{13}{4}) \right) \, dx + \int_{\frac{7}{2}}^2 \left( (x - 3)^2 \left( x - \frac{13}{4} \right) \right) \, dx \)

5. \([-1,2]\)  
b. \( k = -16 \)  
c. \((-\infty, \frac{1}{2})\)  
d. -23

6. \( g(4.5) = 13.5; g'(4.5) = 0; g''(4.5) = -2 \)  
b. \( \frac{29}{8} \)  
c. \( x = 5 \)  
d. \((4.5,13.5)\) and \((9,13.25)\)